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AN OPERATIONAL ANALYSIS
OF SYSTEM CALIBRATION

by

Hasan Basri Mutlu

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Thesis Advisor: Donald P. Gaver

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An Operational Analysis of System Calibration

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I. INTRODUCTION

The effectiveness of many systems depends upon the degree of the calibration of their subsystems. For example, a ship with navigational equipment that is out of calibration may not be able to locate its destination, or, in the case of a Navy ship, locate an adversary. If the Navy ship's weapon system is also out of calibration the difficulties are compounded. An analogous problem arises in connection with engine de-tuning, when fuel consumption will likely increase and performance decrease, and with drift of communication systems. The detrimental effect of mis-calibration is well recognized: Navy ships and other systems are taken to ranges or other facilities for testing and re-calibration.

The purpose of this thesis is to set up mathematical models for studying ways of dealing with mis-calibration. If the various aspects of the problem can be assembled, some guidance is then available for dealing with it effectively. Although various realistic elements of the problem can be introduced, the fundamental issue is this: given that important subsystems depart from calibration and effectiveness as time passes, it is desirable to determine a schedule for re-calibration that (nearly) optimizes system operational effectiveness. Frequent calibration of important systems would be highly desirable if this were a cost-free operation,

but in reality the operational cost of calibration is time--time during which the system is unavailable for, or so degraded as to be incapable of adequately performing, its operational purpose. Figure 1 is an idealized graph of operational effectiveness against time. The periods of duration C denote those periods during which the system has zero effectiveness because it is undergoing calibration and hence is out of the operational area; the periods of duration T represent those periods during which the system is operational, but of diminishing effectiveness.

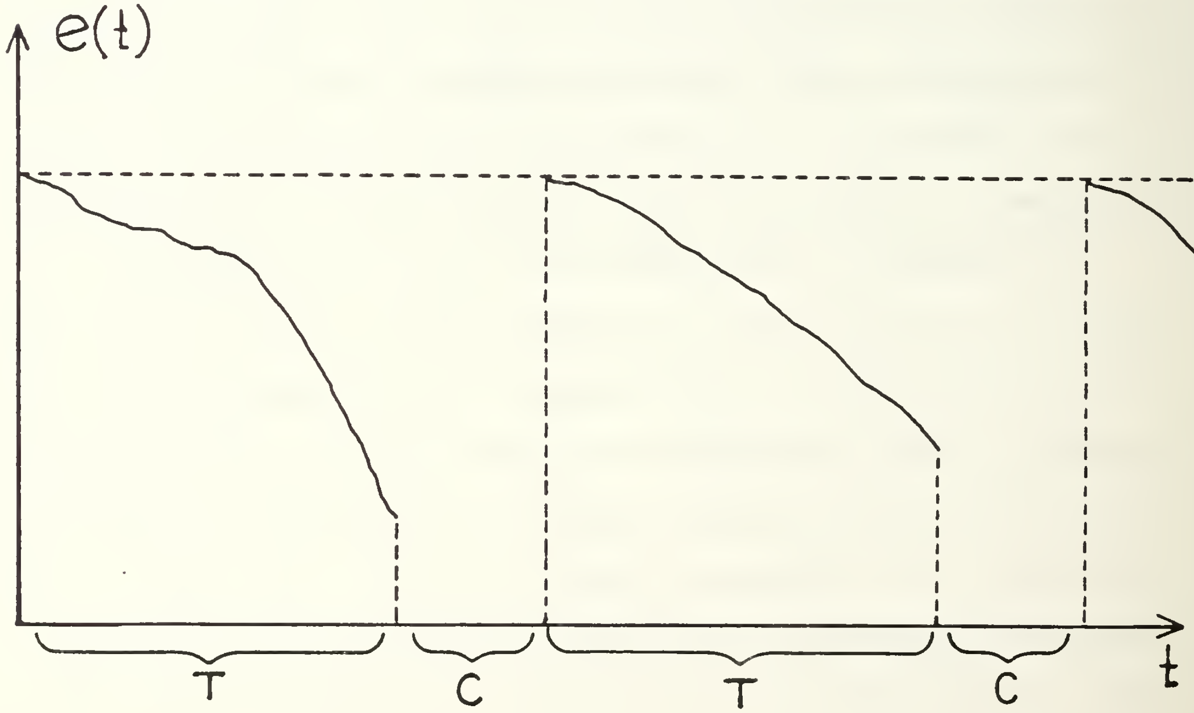


Figure 1.1. Idealized Graph of Operational Effectiveness

The graph suggests that if effectiveness drops with time there will be an optimal value for T, a "best" period, T*, at which to calibrate. We now show how such a period may be determined. Later, more complex and realistic models and simulation results will be introduced.

II. MATHEMATICAL MODELS

For a mathematical treatment let $e(t)$ be the effectiveness, e.g., the probability of successful mission completion, at time t after the calibrated system returns to service. Let C be the time required for calibration, and T the duty or on-station time. Then the average effectiveness over a cycle of length $T+C$, and hence in the long run, is

$$\bar{e}(T) = \frac{\int_0^T e(t)\,dt + 0}{T + C} \; ; \tag{2.1}$$

the term 0 represents and emphasizes the total lack of effectiveness during the calibration period. In order to maximize $\bar{e}(T)$ it is useful to study the derivative

$$\frac{d\bar{e}(T)}{dT} = \frac{(T+C)e(T) - \int_0^T e(t)\,dt}{(T + C)^2} \tag{2.2}$$

as it depends on T: if $d\bar{e}(T)/dT = 0$ for $T^* > 0$ then T^* is a candidate for a time between the end of one calibration and the beginning of the next. Equivalently, (2.2) asks if there is a positive solution T^*, of

$$e(T) = \frac{1}{T+C} \int_0^T e(t)\,dt \tag{2.3}$$

for fixed positive C. The fact that such a solution always exists, and that it defines an optimum can be established from the usual second derivative criterion. Since the optimal T satisfies (2.3), it turns out that at the optimum the average effectiveness over an entire cycle equals the effectiveness at the time the active part of the cycle ends; or symbolically

$$\bar{e}(T^*) = e(T^*) \tag{2.4}$$

where the over-bar signifies the time average of effectiveness over $T^* + C$.

To build understanding, examine some extremely simple specific models.

A. LINEAR EFFECTIVENESS LOSS

Put

$$e(t) = \begin{cases} 1 - at, & 0 \leq t \leq a^{-1} \\ 0 & a^{-1} \leq t . \end{cases} \tag{2.5}$$

so that the downward-sloping parts of the graph of Figure 1.1 are strictly linear. Then (2.3), the equation for optimal $T = T^*$, is

$$1 - aT = \frac{1}{T+C}\left(T - \frac{a}{2} T^2\right), \quad 0 \leq T \leq a^{-1} ; \tag{2.6}$$

It is clear that no value of $T > a^{-1}$ can be optimum. The equation (2.6) simplifies to the quadratic

$$aT^2 + 2aCT - 2C = 0 \tag{2.7}$$

with a single positive solution

$$T^* = -C + \sqrt{C^2 + 2C/a} \tag{2.8}$$

at which the optimum value of effectiveness

$$e(T^*) = \frac{\int_0^{T^*} e(t)\,dt}{T^* + C} = 1 - aT^* = 1 + aC - \sqrt{a^2C^2 + 2aC}$$

$$= (1+aC) - \sqrt{(1+aC)^2 - 1} \tag{2.9}$$

It is interesting that the solution depends only upon the parameter aC the product of calibration drift rate, a, and the length of the re-calibration period, C. For instance, if $aC \to 0$ then effectiveness approaches unity if either the rate of calibration degradation, a, approaches zero, or the calibration time, C, approaches zero, or both, <u>or</u> one approaches zero more rapidly than the other gets large. Alternatively, this shows that equal-effectiveness or a-C tradeoff curves are simple hyperbolas in the (a,C) plane.

The above model is rather crude, but is easy to understand. There follows another model that is more qualitatively appealing.

B. LINEAR DEGRADATION WITH DIFFUSE DAMAGE

Consider next a more specific model for effectiveness, one that relates to damage inflicted on a target after time t has elapsed, and the system has developed an (unsuspected) bias of magnitude at. At that time the x-y error made in locating a target is assumed to be given by the joint Gauss/normal density

$$f(x,y;t) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{1}{2}\frac{(x-at)^2}{\sigma^2} - \frac{1}{2}\frac{(y-at)^2}{\sigma^2}\right] \tag{2.10}$$

If a cookie-cutter damage function with radius R is in effect (no damage if $x^2 + y^2 > R^2$, destruction if $x^2 + y^2 \leq R$) then

$$e(t) = \iint\limits_{(x^2 + y^2 \leq R^2)} f(x,y;t)\,dx\,dy \ .$$

However, this is difficult to work with, and even overly simplistic. Instead, suppose that a von Neumann-Gauss diffuse damage function can be used; i.e., that the probability of critical damage to a target located at (0,0) by a weapon with impact point (x,y) is equal to $\delta(x,y) = \exp(-\alpha(x^2 + y^2))$. Then

$$e(t) = \iint \delta(x,y) f(x,y;t)\,dxdy$$

$$= \frac{1}{2\pi\sigma^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[-\alpha(x^2+y^2)]\, f(x,y;t)\,dxdy$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp[-\frac{1}{2}\frac{(at-x)^2}{\sigma^2}] \exp[-\alpha x^2]\,dx\right)^2 \qquad (2.11)$$

by virtue of the symmetry assumed; almost free of charge we can consider asymmetrical damage functions, but the opportunity is declined. The above integral is evaluated at sight: it is seen to be essentially the convolution of two normal densities. After squaring, as demanded by (2.11),

$$e(t) = \frac{1/2\alpha}{(\sigma^2 + 1/2\alpha)} \exp[-\frac{(at)^2}{(\sigma^2 + 1/2\alpha)}] \qquad (2.12)$$

Instead of dropping off linearly, as in the previous case, e(t) first diminishes rather slowly, later falling quite rapidly (exponentially fast) towards zero: by the time $at = \sqrt{\sigma^2 + 1/2\alpha}$, effectiveness is just below 40% of its maximum, while if $at = 0.5\sqrt{\sigma^2 + 1/2\alpha}$, effectiveness is about 78% of the maximum; finally if $at = 0.25\sqrt{\sigma^2 + 1/2\alpha}$, effectiveness is 94% of the maximum. Note that the maximum effectiveness is $(1 + 2\alpha\sigma^2)^{-1} \leq 1$; if either σ^2 or α become large, meaning that if either weapon effectiveness falls off rapidly with miss distance (α large) <u>or</u> the ultimate weapon

delivery variance is great (σ^2 large), then even maximum effectiveness is low.

In order to solve for the optimum T* write

$$(T+C)\exp(-(aT)^2/(\sigma^2+1/2\alpha)) = \int_0^T \exp(-(at)^2/(\sigma^2+1/2\alpha))dt . \tag{2.13}$$

Change the variables to the dimensionless version

$$\tau = (aT)/(\sigma^2+1/2\alpha)^{1/2} \; ; \quad \gamma = (aC)/(\sigma^2+1/2\alpha)^{1/2} , \tag{2.14}$$

so one can solve the following dimensionless equation once and for all for τ^*:

$$(\tau+\gamma)\exp(-\tau^2) = \int_0^\tau \exp(-z^2)dz \; ; \tag{2.15}$$

the positive value of τ, namely τ^*, that satisfies this equation may be located by Newton-Raphson, or even graphically: one can plot, for given γ,

$$L(\tau) = (\tau+\gamma)\exp(-\tau^2)$$

and

$$R(\tau) = \int_0^\tau \exp(-z^2)dz$$

on the same piece of paper, vs. τ.

The arbitrary selected γ values and the corresponding τ^* values from the computer program which solves the dimensionless equation (2.15) are shown in Table 1.

TABLE 1

Gamma and Optimum Tau Values

GAMMA	TAU
0.001	0.025
0.002	0.115
0.003	0.144
0.004	0.165
0.005	0.181
0.006	0.195
0.007	0.207
0.008	0.218
0.009	0.227
0.01	0.236
0.02	0.302
0.03	0.346
0.04	0.381
0.05	0.410
0.06	0.434
0.07	0.456
0.08	0.476
0.09	0.494
0.1	0.511
0.2	0.632
0.3	0.713
0.4	0.775
0.5	0.825
0.6	0.867
0.7	0.904
0.8	0.937
0.9	0.966
1.0	0.992

In order to redefine τ and γ in (2.14) and the effectiveness formula (2.12) in more meaningful form, again change variables to

$$v = 1/2\alpha \; ; \quad p = \frac{1/2\alpha}{\sigma^2 + 1/2\alpha} \; ; \quad k = a/\sigma \qquad (2.16)$$

where v and p might be called <u>vulnerability</u> and <u>probability of success</u> respectively, and k is constant. In the simulation chapter connections are developed between v and the radius of a (roughly) equivalent cookie-cutter damage function. So one can write

$$\tau = k\sqrt{1-p}\, T \; ; \qquad \gamma = k\sqrt{1-p}\, C \; . \qquad (2.17)$$

We focus attention on the representation (2.12) in what follows, mainly for analytical and computational convenience.

$$e(t) = p \exp\left[-(kt)^2(1-p)\right] \qquad (2.18)$$

Thus, the preceding expression at optimum leads to the relationship

$$e(T^*) = p \exp(-\tau^*)^2 \qquad (2.19)$$

and consequently, the optimal <u>proportion of on-station time</u> can be obtained as follows:

$$\frac{\tau^*}{\gamma} = \frac{T^*}{C}$$

$$T^* = \frac{\tau^*}{k\sqrt{1-p}} , \qquad (2.20)$$

so the proportion of on-station time is, under optimum conditions,

$$\frac{T^*}{T^* + C} = \frac{\tau^*}{\tau^* + \gamma} . \qquad (2.21)$$

Since optimum τ values are available from Table 1, one can very easily calculate the effectiveness given some constant variance and v or only p. Some of the results are tabulated in Table 2 as an example.

Additionally, plots of tau vs. gamma and effectiveness vs. tau are presented in Appendix A. It is observed that the effectiveness decreases as the variance increases while v is held constant. Effectiveness vs. tau plots illustrate the behavior of the effectiveness representation in a more understandable fashion than does the formula itself.

Example: Suppose $a = 1.5$ yds/month, $C = 0.5$ month, $\sigma^2 = 20$ $(\text{yds})^2$ and $p = 0.9$ are given. First find γ from (2.17) as 0.053, then look up corresponding τ^* value from Table 1 which is 0.417. Later, from (2.20) T^* is 3.93 months and from (2.21) the proportion of on-station time is 88.7%, and from (2.19) or Table 1 an average effectiveness of 75.6% can be obtained.

TABLE 2

Effectiveness for Constant Variance and v

EFFECTIVENESS ($\sigma^2 = 10$; $v = 200$)	EFFECTIVENESS ($\sigma^2 = 20$; $v = 200$)
0.952	0.908
0.940	0.897
0.933	0.890
0.927	0.885
0.922	0.880
0.917	0.875
0.912	0.871
0.908	0.867
0.904	0.863
0.901	0.860
0.869	0.830
0.845	0.806
0.824	0.786
0.805	0.768
0.789	0.753
0.773	0.738
0.759	0.725
0.746	0.712
0.733	0.700
0.639	0.610
0.573	0.547
0.522	0.499
0.482	0.460
0.449	0.429
0.421	0.401
0.396	0.378
0.374	0.357
0.356	0.340

C. LINEAR DEGRADATION WITH DIFFUSE DAMAGE USING RANDOM DRIFT

For an alternative model, that incorporates the possibly different drift rates of different individual ships or systems, suppose that the drift, $\underset{\sim}{a}$, is a random variable with an appropriate distribution function instead of a constant as in (2.12), namely, the effectiveness conditional on $\underset{\sim}{a}$ is

$$e(t;\underset{\sim}{a}) = \frac{1/2\alpha}{(\sigma^2 + 1/2\alpha)} \exp\left[- \frac{(\underset{\sim}{a}t)^2}{(\sigma^2 + 1/2\alpha)}\right] . \qquad (2.22)$$

Then, the expected average or unconditional effectiveness over a cycle of length T+C, in the long run, becomes

$$E[\bar{e}(T;\underset{\sim}{a})] = \frac{\int_0^T E[e(t;\underset{\sim}{a})]dt}{T + C} \qquad (2.23)$$

in order to be specific (but not necessarily realistic) and also so that explicit mathematical results are obtained, let $\underset{\sim}{a}^2$ have a gamma distribution function with parameters λ and β,

$$f\underset{\sim}{a}^2(x;\lambda,\beta) = \frac{\lambda e^{-\lambda x}(\lambda x)^{\beta-1}}{\Gamma(\beta)} \quad \lambda,\beta > 0 , \qquad (2.24)$$

and put for fixed a^2, i.e., the square of the drift rate away from calibration,

$$E(\underset{\sim}{a}^2) = a^2 = \frac{\beta}{\lambda} \; ; \quad \text{so} \quad \text{Var}(\underset{\sim}{a}^2) = \frac{\beta}{\lambda^2} = \frac{a^4}{\underset{\sim}{\beta}} \tag{2.25}$$

now use (2.22) and (2.23) to obtain

$$E[e(t;\underset{\sim}{a})] = \frac{1/2\alpha}{(\sigma^2 + 1/2\alpha)} \int_0^\infty \exp[-x(\frac{t^2}{\sigma^2 + 1/2\alpha})] f\underset{\sim}{a}^2(x;\lambda,\beta)dx$$

after substituting the gamma density function it is easily seen that the result of the integration yields

$$E[e(t;\underset{\sim}{a})] = \frac{1/2\alpha}{(\sigma^2 + 1/2\alpha)} \left(\frac{\lambda}{\lambda + \dfrac{t^2}{\sigma^2 + 1/2\alpha}} \right)^{\beta} , \tag{2.26}$$

or equivalently, in view of (2.25)

$$E[e(t;\underset{\sim}{a})] = \frac{1/2\alpha}{(\sigma^2 + 1/2\alpha)} \left(\frac{1}{1 + \dfrac{(at)^2}{(\sigma^2 + 1/2\alpha)\beta}} \right)^{\beta} \tag{2.27}$$

Various analytical properties of the previously described model will now be recorded. These provide useful insights into the behavior of the effectiveness at time t.

1. If $\beta \to \infty$, (2.27) becomes

$$E[e(t;\underset{\sim}{a})] = \frac{1/2\alpha}{\sigma^2 + 1/2\alpha} \exp[-(at)^2] \tag{2.28}$$

which reflects the fact that if β increases the variance of $\tilde{a}$ in the distribution of drift rate decreases towards zero, and the situation reduces to that of Model B.

2. If $\beta \to 1$, then

$$E[e(t;\tilde{a})] = \frac{1/2\alpha}{(\sigma^2 + 1/2\alpha + (at)^2)} , \tag{2.29}$$

which is larger than the effectiveness in the equal-drift case.

In order to solve for the optimum T* for the general case write

$$\frac{T+C}{\left(1 + \frac{(aT)^2}{(\sigma^2 + 1/2\alpha)\beta}\right)^{\beta}} = \int_0^T \frac{1}{\left(1 + \frac{(at)^2}{(\sigma^2 + 1/2\alpha)\beta}\right)^{\beta}} \, dt . \tag{2.30}$$

Change the variables to

$$\tau = (aT)/(\sigma^2 + 1/2\alpha)^{1/2} ; \quad \gamma = (aC)/(\sigma^2 + 1/2\alpha)^{1/2} \tag{2.31}$$

so one can solve the dimensionless equation

$$\frac{\tau + \gamma}{(1 + \frac{\tau^2}{\beta})^{\beta}} = \int_0^{\tau} \frac{dz}{(1 + \frac{z^2}{\beta})^{\beta}} ; \tag{2.32}$$

the positive value of τ, namely τ^*, that satisfies this equation for any constant β may be found by a computer

program. In fact, one may get the solution for the special case $\beta = 1$ by making use of arctg integration for the right-hand side:

$$\gamma = (\text{arctg}\ \tau)(1 + \tau^2) - \tau . \tag{2.33}$$

In general, the right-hand integral can be transformed to the integral of a Student's t density, and the t-tables found in most statistics books can be used to evaluate it.

Again, the arbitrary selected γ values and the corresponding τ^* values for $\beta = 1$ are presented in Table 3.

At this point, it is very easy to calculate the effectiveness given some constant variance and v or only p from (2.19). Some of the results are listed in Table 4 as an example.

In addition to the tables, plots of tau vs. gamma and effectiveness vs. tau are presented in Appendix B. As in the previous case, effectiveness decreases as the variance increases when v is held constant.

TABLE 3

Gamma and Optimum Tau Values (β = 1)

GAMMA	TAU
0.001	0.025
0.002	0.115
0.003	0.145
0.004	0.166
0.005	0.182
0.006	0.196
0.007	0.209
0.008	0.220
0.009	0.230
0.01	0.239
0.02	0.307
0.03	0.355
0.04	0.392
0.05	0.424
0.06	0,451
0.07	0.476
0.08	0.499
0.09	0.520
0.1	0.539
0.2	0.687
0.3	0.793
0.4	0.879
0.5	0.952
0.6	1.018
0.7	1.077
0.8	1.131
0.9	1.181
1.0	1.228

TABLE 4

Effectiveness for Constant Variance and v (β = 1)

EFFECTIVENESS ($\sigma^2 = 20$; $v = 150$)	EFFECTIVENESS ($\sigma^2 = 30$; $v = 150$)
0.882	0.833
0.871	0.822
0.864	0.816
0.859	0.811
0.854	0.807
0.850	0.802
0.845	0.798
0.842	0.795
0.838	0.791
0.835	0.788
0.806	0.761
0.784	0.740
0.765	0.722
0.748	0.706
0.733	0.692
0.719	0.679
0.706	0.667
0.694	0.656
0.684	0.646
0.599	0.566
0.542	0.512
0.498	0.470
0.463	0.437
0.433	0.409
0.408	0.386
0.387	0.366
0.368	0.348
0.352	0.332

III. TRANSFORMATIONS AND SIMULATION

A. TRANSFORMATIONS

Earlier it has been shown that τ^* can be computed in terms of γ. In order to simplify this step, it would be desirable to be able to represent τ^* by some simple formula in terms of γ. Following the lead of statistical regression studies, it is sometimes possible to investigate the effects produced by transformations of the predictor variables, or by transformations of the response variable, or by both. Clearly there are many possible transformations of gamma and tau values. Several different transformations of gamma and tau could be tried for the same model, of course. The choice of which is sometimes difficult to decide and the choice would often be made on the basis of previous knowledge of the gamma and tau under study. The purpose of making transformations of this type is to be able to use a simple regression model in the transformed tau and gamma, rather than a more complicated one in the original gamma and tau. Some suitable transformations of gamma or tau can also be found by plotting them in various ways. First, ln(tau) vs. ln(gamma) has been plotted for the linear degradation with diffuse damage case and the linear degradation with diffuse damage using random drift case, later various power transformations have been applied to tau values and simple

regression equations have been derived in order to obtain the optimum tau values directly for arbitrary selected gamma values without having to go to tables or equations (2.15) and (2.32). Some of the transformation plots are shown in Appendix C. After obtaining a suitable power transformation of tau, one could guidely calculate the effectiveness values by using predicted tau values from regression equation. This attempt at simplification deserves more study before it can be said to be truly satisfactory.

B. SIMULATION

Simulation is essentially a controlled statistical sampling technique (experiment) which is used, in conjunction with a model, to obtain approximate answers for complex (probabilistic) problems when analytical and numerical techniques are too expensive, or infeasible.

The main purpose of the simulation in this thesis is to be able to evaluate effectiveness for other kinds of damage functions or error distributions which are difficult to work with, as alternatives to a von Neumann-Gauss diffuse damage function. For example, if a cookie-cutter damage function with radius R is in effect then a closed form solution of the effectiveness similar to (2.12) is not as simple. Unlike a mathematical solution, the answer one obtains from a simulation is an estimate of the effectiveness. It is absolutely necessary to have some idea of the precision of the effectiveness. For this reason, the effectiveness

estimated from each simulation has error bounds of two standard deviations for valid comparisons. The interactive simulation program, written in APL, is presented in Appendix D.

The scenario developed in this program determines the optimal time for a submarine to come in to port for instrument re-calibration. In other words, simulation is being used to determine T* for a variety of cases--ones in which the previous neat mathematical theory of Chapter II cannot easily be extended. A vector of possible times (in arbitrary time units such as days) at which the submarine should be brought back for equipment re-calibration is needed. For each of these times the program estimates the expected effectiveness of the submarine. The time that corresponds to maximal effectiveness is considered optimal. Although the effectiveness of the submarine changes continuously with time, in the simulation the effectiveness is estimated only at discrete but closely-spaced time points. The more points one has, the smoother the effectiveness curve, but the longer the program takes to run. In addition, the duration of re-calibration of the equipment (C) and the number of replications of the simulation should be entered. Again, the precision of the estimates of the effectiveness curve gets better with more replications, but, again, it takes longer to run the program.

Effectiveness is measured as the probability of damaging a target ship that is 1000 distance units away from the

submarine. The weapon is a straight-run classical torpedo with a proximity fuse. The submarine fires the torpedo along some bearing and the torpedo is supposed to explode at the point nearest to the target. But the equipment to locate the target develops calibration problems with time, namely calibration drifts by a certain distance for every time unit according to the following specific alternative model options, any one of which may be considered by the analyst:

1. It might get deterministically worse with time. So, the expectation of drift becomes E(at) = at; the rate a must be specified. This is Model 1.

2. It might get randomly worse with time. On day T, the drift is mismeasured by an amount T x Normal(0,Sigma). Although the mean error is zero, the variance of the error increases as $\text{sigma}^2 \times \text{time}$. Notice that the random multiplier is constant in each replication of the simulation. This is Model 2.

3. It might fluctuate randomly with time. On each day, the drift is mismeasured by an additional drift error. This error term is random and comes from a Normal(0,Sigma) distribution, where sigma is expressed in distance units and represents the standard deviation of the error distribution. Notice that the expected error is always zero, although the variance grows proportionally with time. In this case, calibration can improve or worsen with time. This is Model 3.

4. It might exhibit a random drift, with magnitude drawn from a gamma distribution. Thus, calibration drift for every time unit becomes a gamma random variable with shape parameters lambda and beta; the drift gets worse with time. Note that the program uses GAMMACK APL library function to generate the incomplete gamma random variable. This is Model 4. The gamma variability explains the differences in drift exhibited by different system copies.

The user may choose which of the above models best describes his or her situation.

After a straight-run classical torpedo is aimed at a point influenced by one of the preceding calibration errors, it may not explode at precisely the point on that bearing that is closest to the target, i.e., the proximity fuse is assumed to be not perfectly accurate, as is true in reality. The error between the closest point and the explosion point can come from either normal distribution with variances in the X and Y direction or a uniform distribution with (-X,X) and (-Y,Y).

Finally, the target is damaged with a probability calculated according to one of the following optional functions:

1. Gauss diffuse damage: Probability of critical damage to a target located at (0,0) by a weapon with impact point (x,y) is equal to $\delta(x,y) = \exp(-\alpha(x^2 + y^2))$.

2. Cookie-cutter: The torpedo will destroy the target if it is within a certain radius R, and it will do no damage if it is outside this radius.

3. Trapezoidal: The trapezoidal damage function has a central, circular plateau of radius R_1. If the target is within R_1 distance units from the exploding torpedo, then the target is damaged with probability one as it is in the cookie-cutter. In addition, the function has an outer circular rim, of radius R_2, $R_2 > R_1$, beyond which the probability of damaging the ship is zero. Between the two radii, the damage probability goes down linearly.

As a result, the simulation program provides three basic output arrays. EFF contains the estimated effectiveness at each time increment, delta, out to the maximum time. Again, the effectiveness is simply the probability of the torpedo destroying the target. STDEFF contains the standard deviations of the estimates in EFF. Lastly, AVGEFF contains the long-term average effectiveness of the submarine if it returns after the various times following calibration delays input at the beginning of the program.

Plots of simulation results in Appendix E were obtained from the general plot function in GRAFSTAT which is an APL workspace for the interactive creation of scientific-engineering graphics, for interactive data analysis and for the interactive development of APL graphics output routines. It runs on both the IBM 3277GA graphics terminal and the 3278/79 terminal. Full color control is available when running on the 3279 terminal. The simulation program is attached to GRAFSTAT in order not to waste time in assessment of the

results and to be able to make sensitivity analyses easily. For each run, two different graphs are obtained. One of them shows estimated effectiveness of a submarine with error bounds of two standard deviations. The other one demonstrates the long-term average effectiveness of a submarine, given that it stays at sea for a tour of length T. Since there is a large number of combinations in the program, only limited numbers of results are presented in this thesis. The first six plots simply exhibit the sensitivity of the system effectiveness to the parameter alpha of the Gauss-diffuse damage function; other variables are held constant. It is observed that the optimum T gets smaller as the parameter alpha gets bigger; since a large α corresponds to a small effective damage region, a short tour length is necessary to keep effectiveness high. In addition, some interesting combinations of alternatives are also presented in order to give an idea about the behavior of the other parameters. There is not much to say about them since they are quite self-explanatory.

It is of interest to check out the possible relationship between Gauss diffuse and cookie-cutter damage functions. An analytical-numerical solution for the optimum interval, T^*, is available for the von Neumann-Gauss function, but none is for the cookie-cutter or the trapezoidal functions. If the latter can be reasonably matched to the former, an approximate analytical solution is available for these

latter damage functions. Possible matches to cookie-cutter are these:

1. Mean matching: $\int_0^\infty r\exp(-\alpha r^2)dr = 1/2\alpha = \int_0^R r dr = R^2/2$.

So, the Gauss diffuse damage with parameter α and cookie-cutter with radius R are approximately equivalent.

2. Median matching: $\exp(-\alpha r^2) = 0.5$ which is equivalent to $\alpha(R) = \ln(2)/R^2$. Thus, this transformation matches the medians of the two functions.

It is not possible to find a unique best transformation between α and R for all cases. However, one can test a proposed transformation for each specific case by simulation. For demonstration purposes, $\alpha(R) = \ln(5/3)/R^2$ is a proper transformation for R = 14.29 on condition that the other variables are held constant. It provides the same optimum T for Gauss diffuse and cookie-cutter damage functions as is seen in Appendix E.

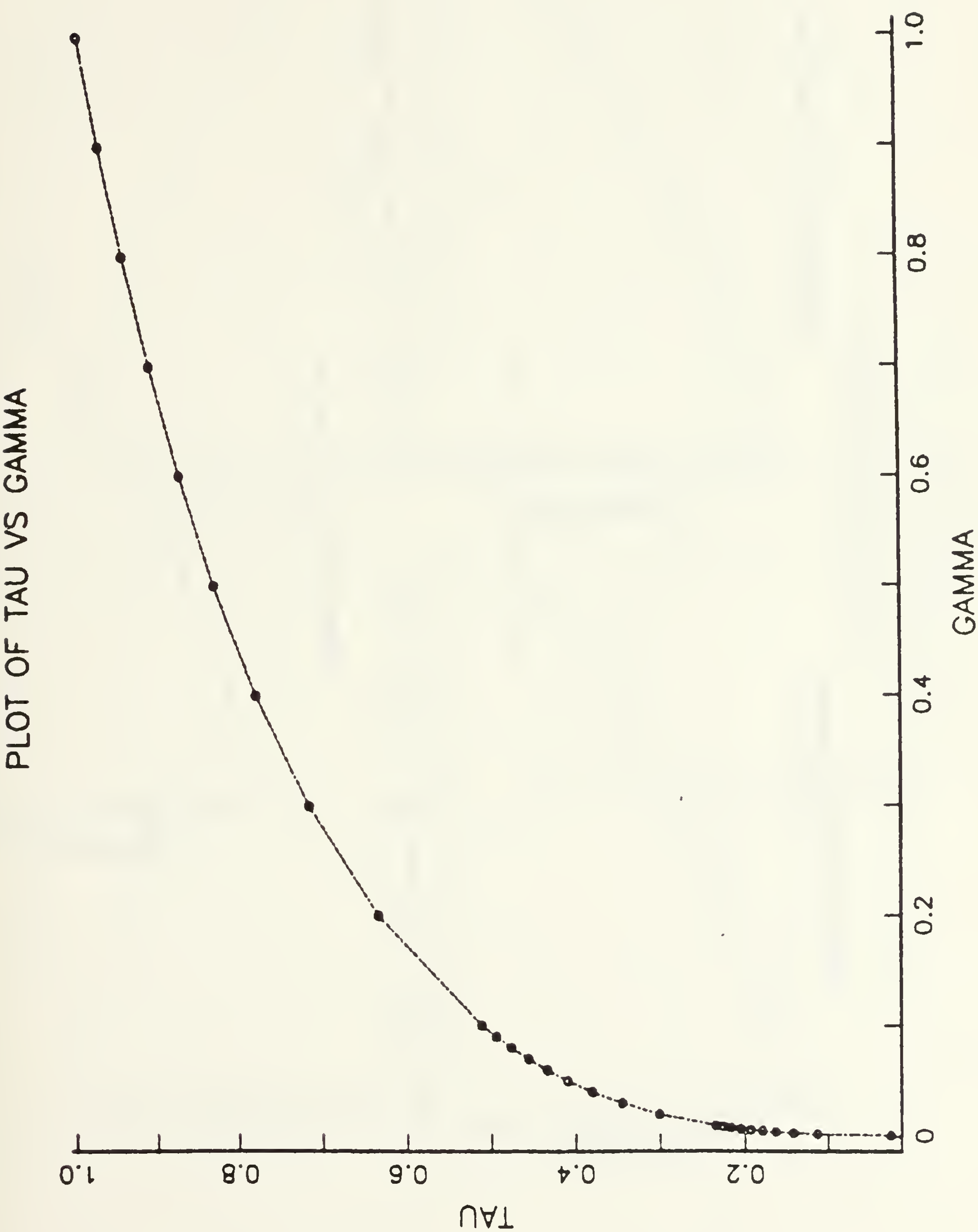
IV. CONCLUSION

The purpose of this thesis has been to show that mathematical models, augmented by a computer simulation program, can provide useful ways of studying the impact of miscalibration upon operational effectiveness. We have concentrated here on specific and convenient models, but it is obvious that other mathematical models can be treated similarly. Other analyses similar to the ones we have discussed in the computer program can be conducted pertaining to the other alternatives. The relative effectiveness of different system configurations can also be investigated. The results of the simulation can then be analyzed to ascertain the significance of different factors in various scenarios.

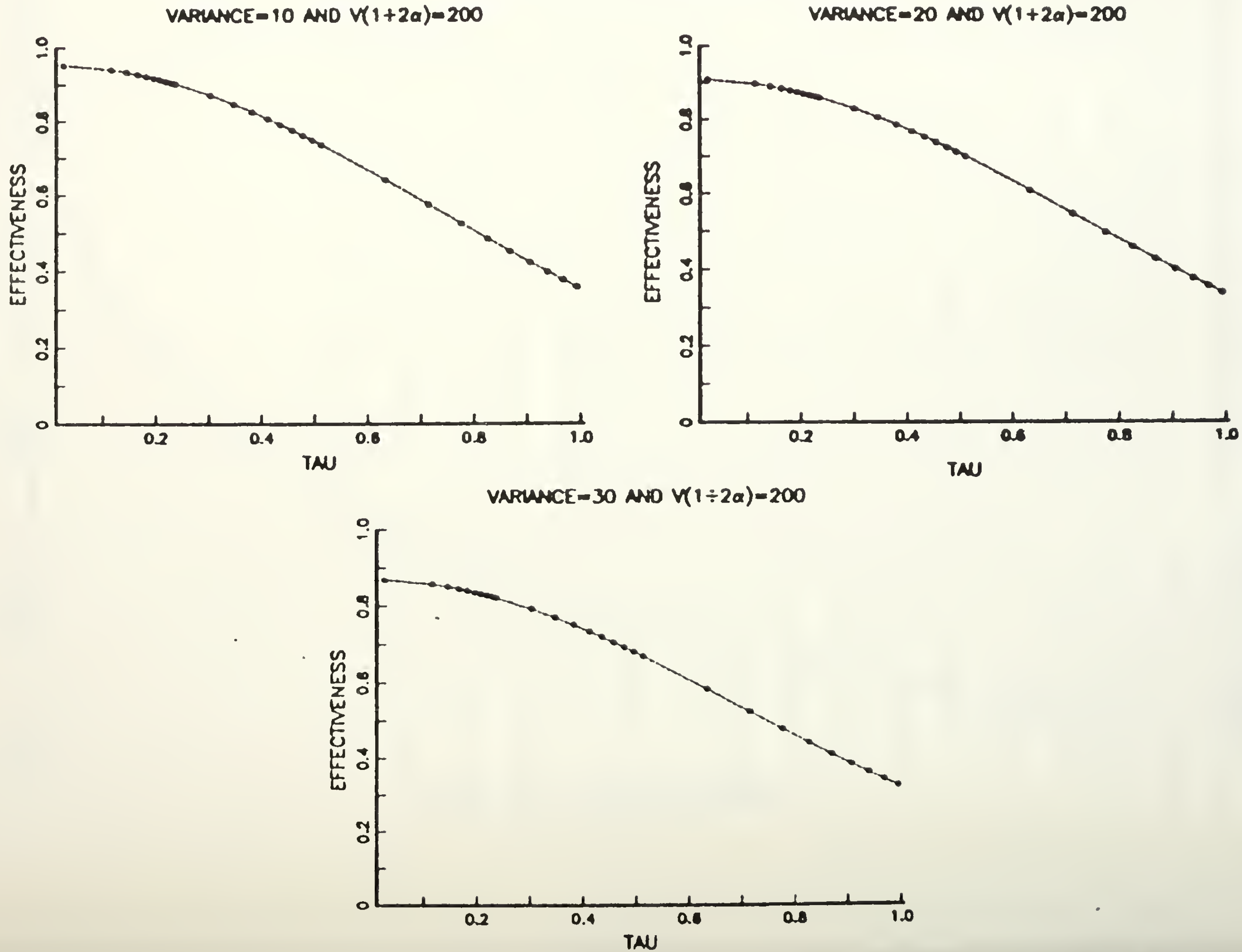
The possibility exists that operational data will reveal different underlying distributions, and suggest alternatives for evaluating effectiveness other than the ones described in this thesis. The present thesis is to be considered a pilot study of the calibration issue.

APPENDIX A

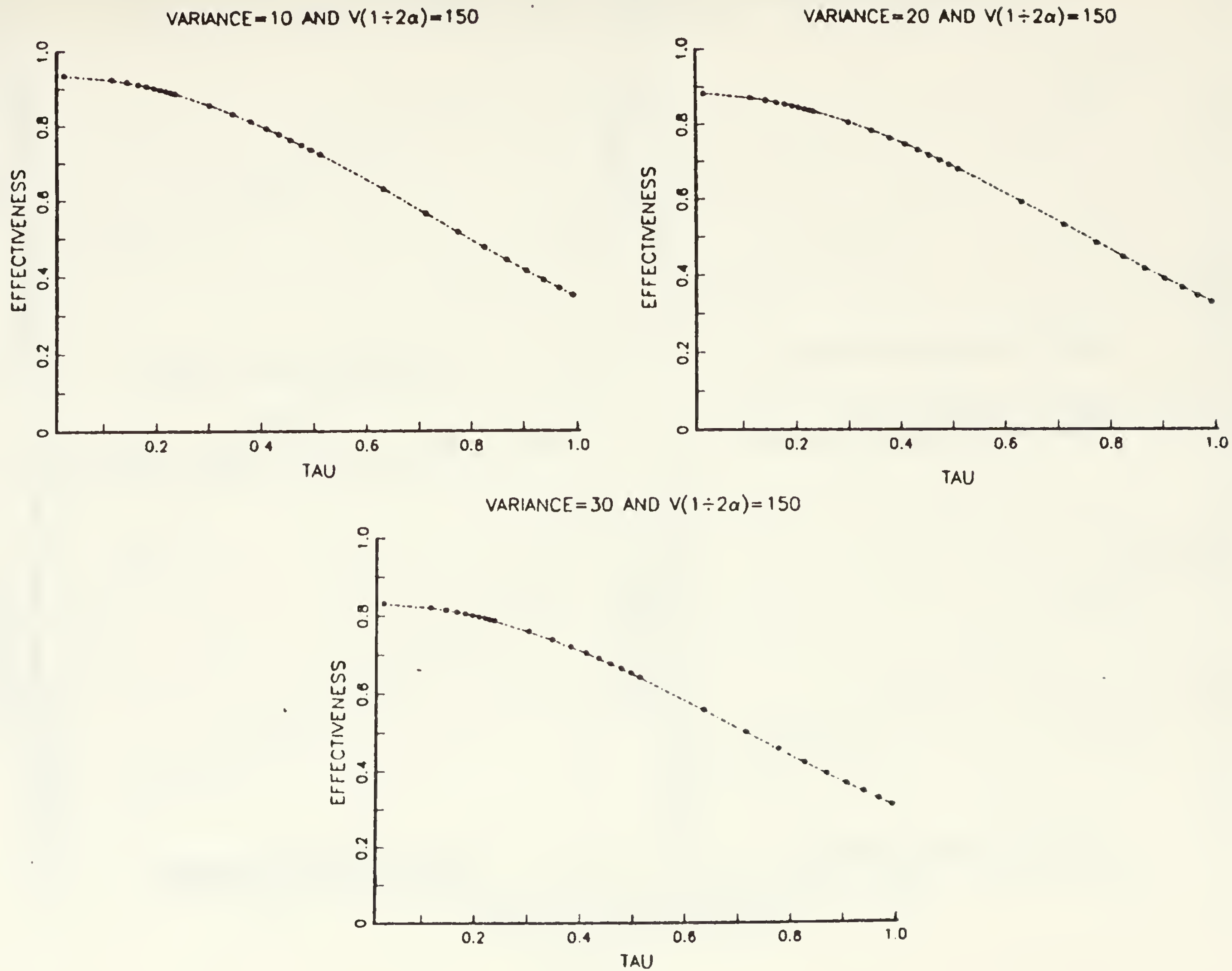
PLOTS OF TAU VS. GAMMA AND EFFECTIVENESS VS. TAU



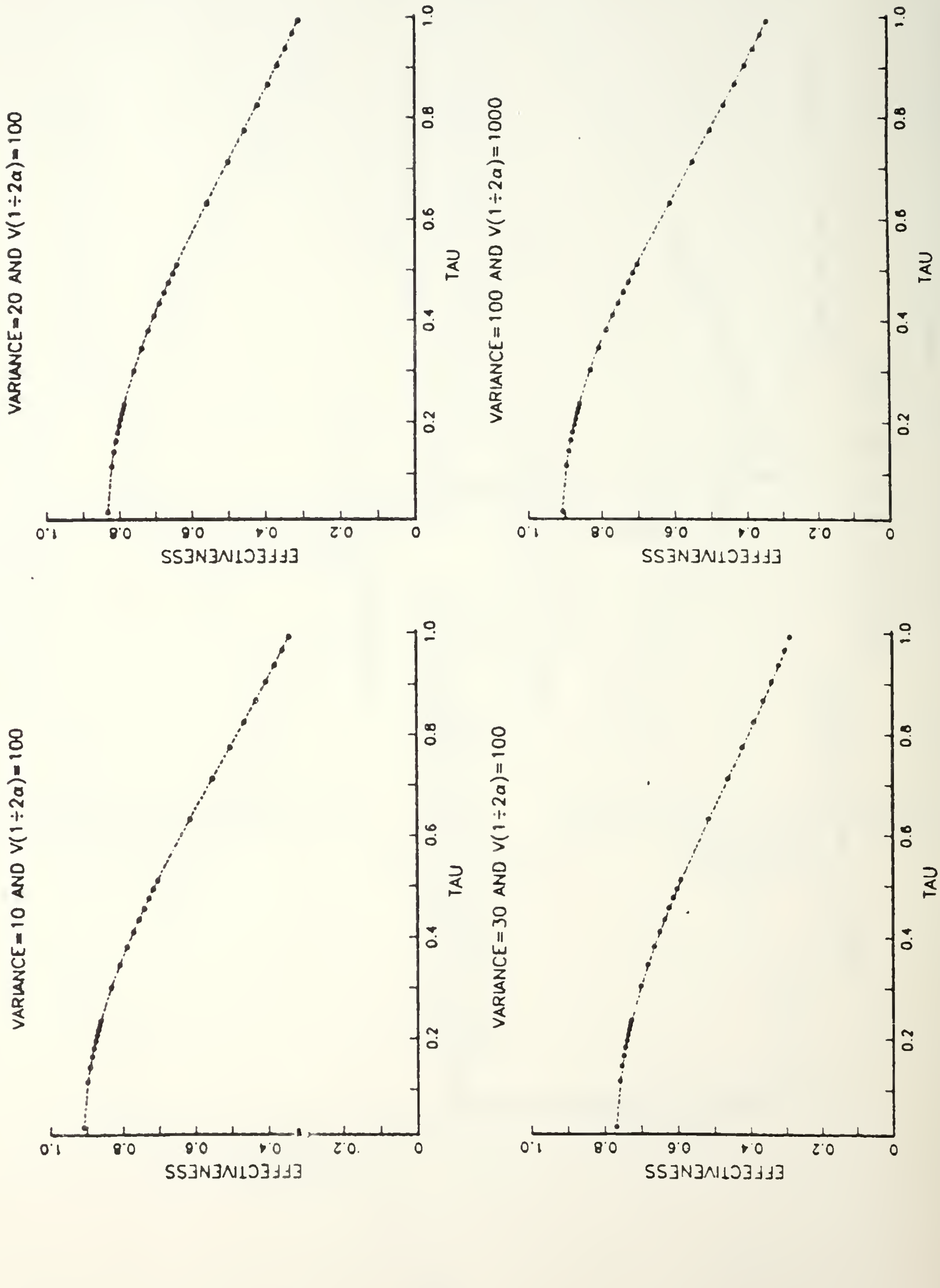
PLOT OF E(T) VS TAU



PLOT OF E(T) VS TAU

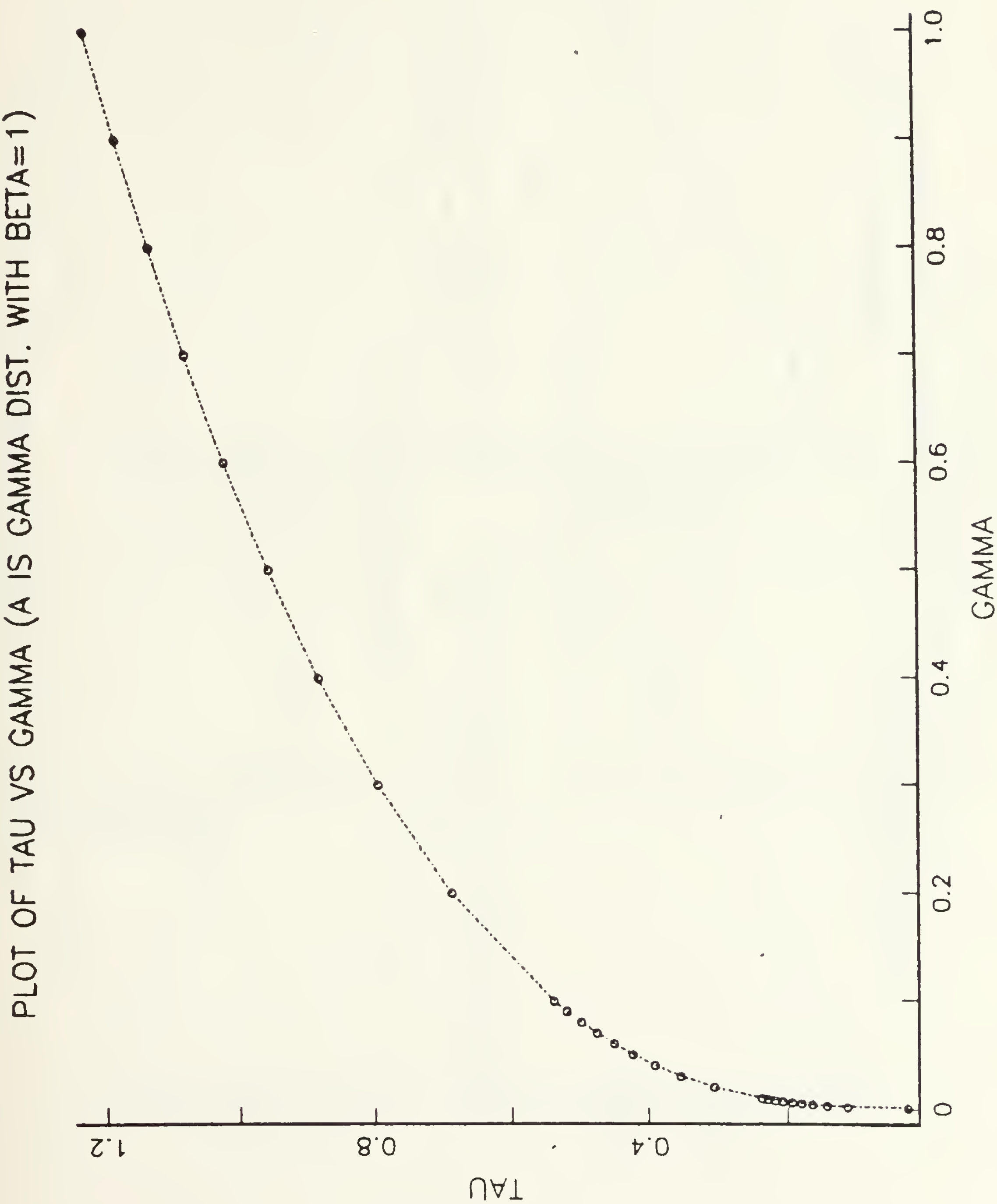


PLOT OF E(T) VS TAU



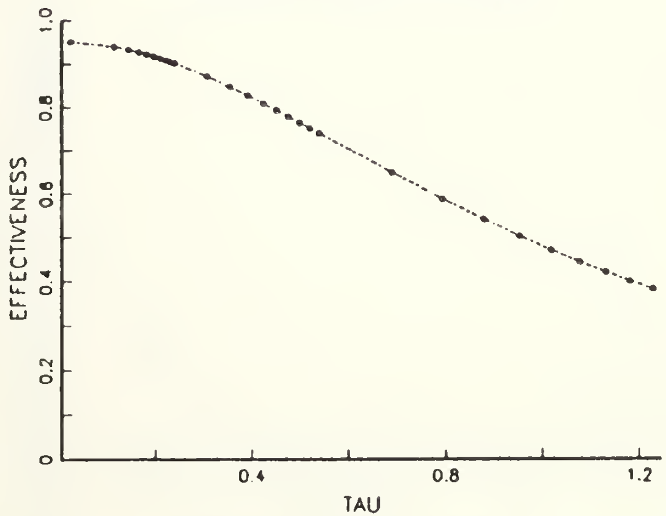
APPENDIX B

PLOTS OF TAU VS. GAMMA AND EFFECTIVENESS VS. TAU (BETA = 1)

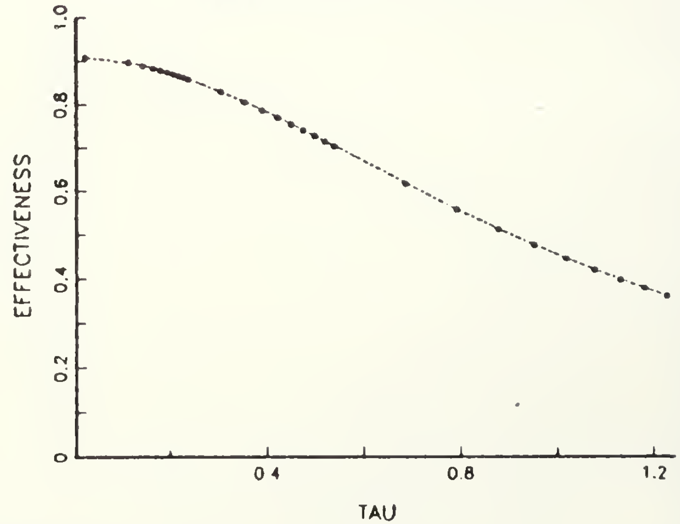


PLOT OF E(T) VS TAU (A IS GAMMA DIST. WITH BETA=1)

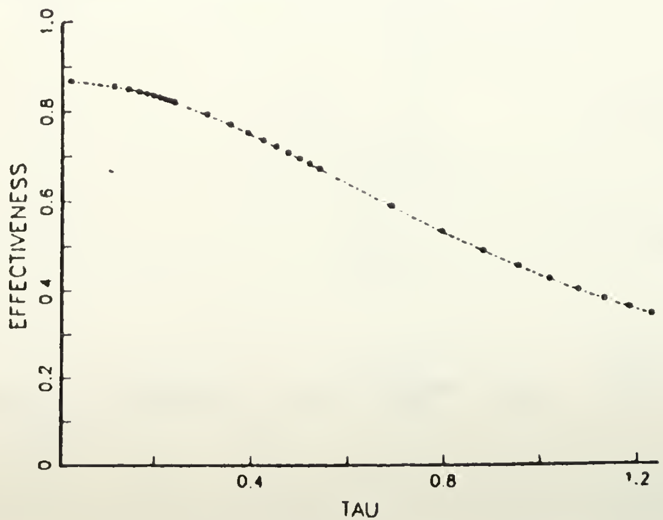
VARIANCE=10 AND V(1÷2α)=200



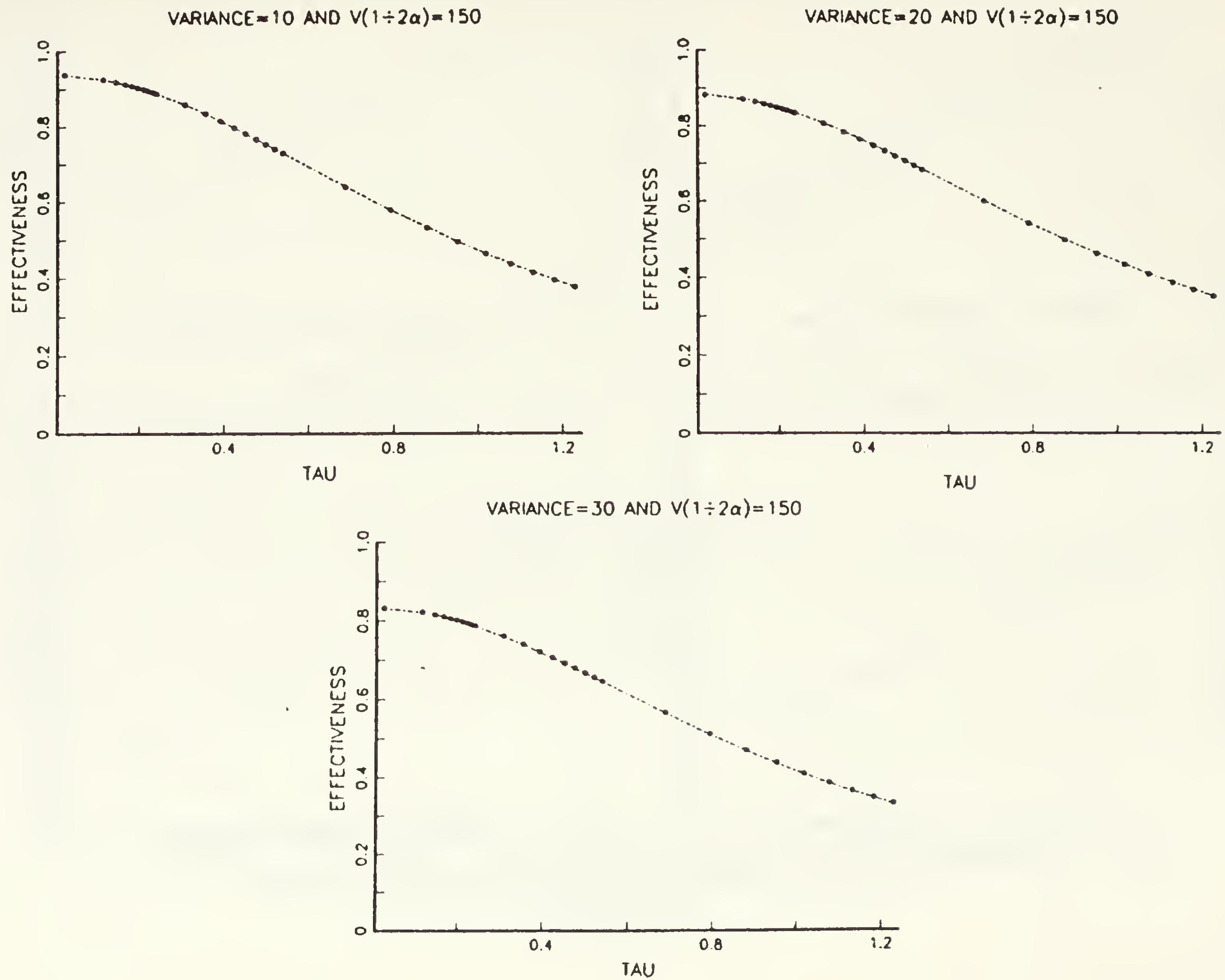
VARIANCE=20 AND V(1÷2α)=200



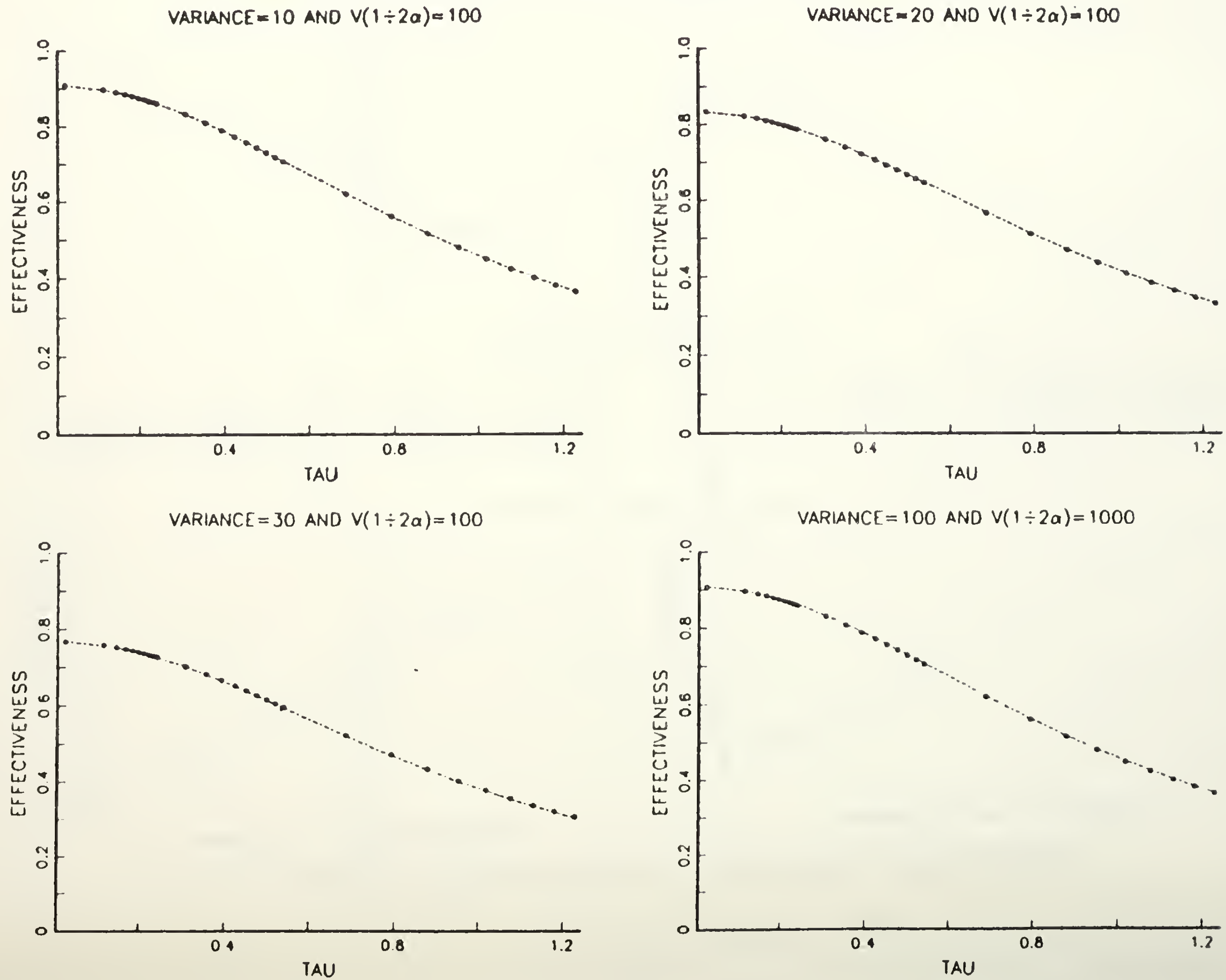
VARIANCE=30 AND V(1÷2α)=200



PLOT OF E(T) VS TAU (A IS GAMMA DIST. WITH BETA=1)

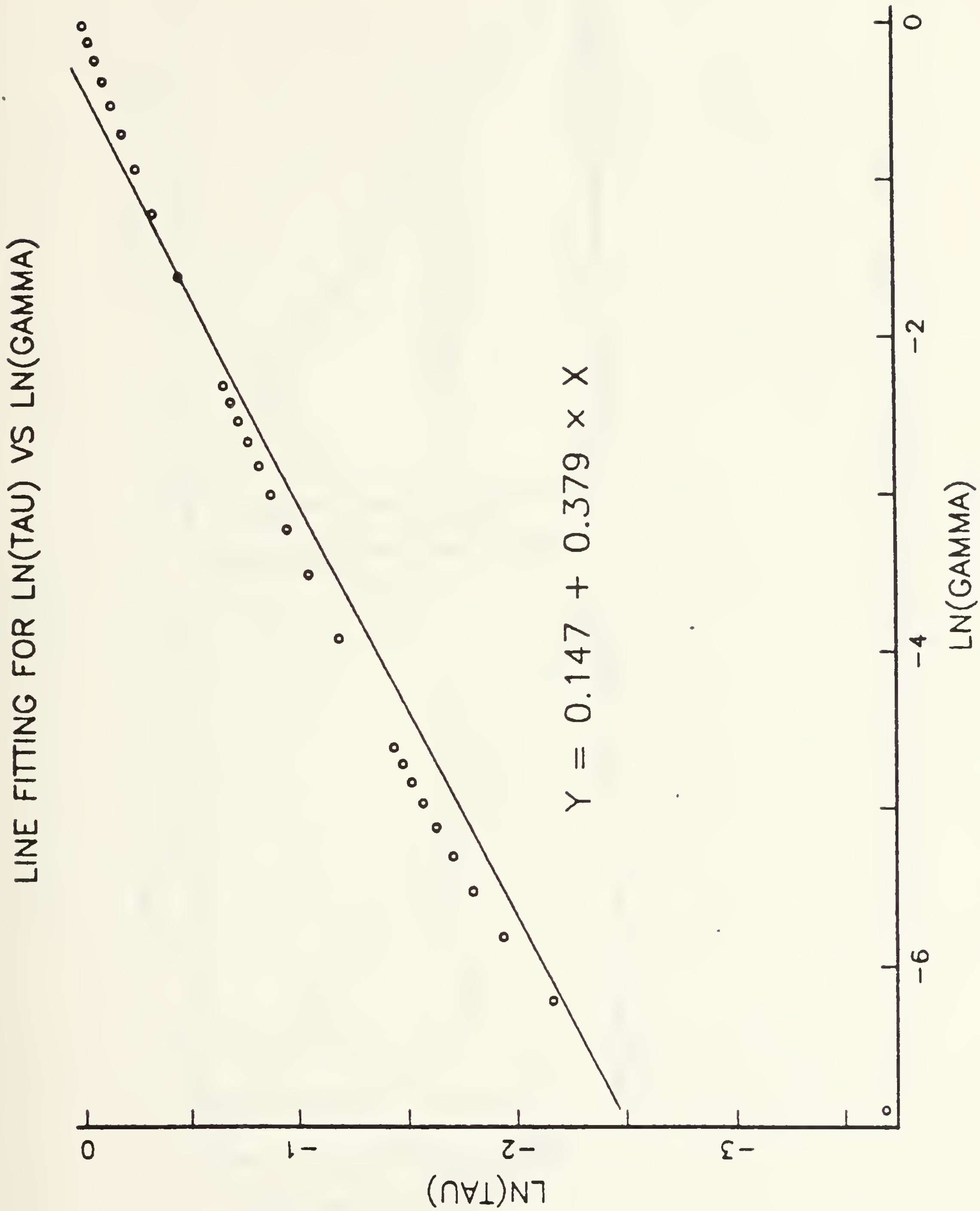


PLOT OF E(T) VS TAU (A IS GAMMA DIST. WITH BETA=1)

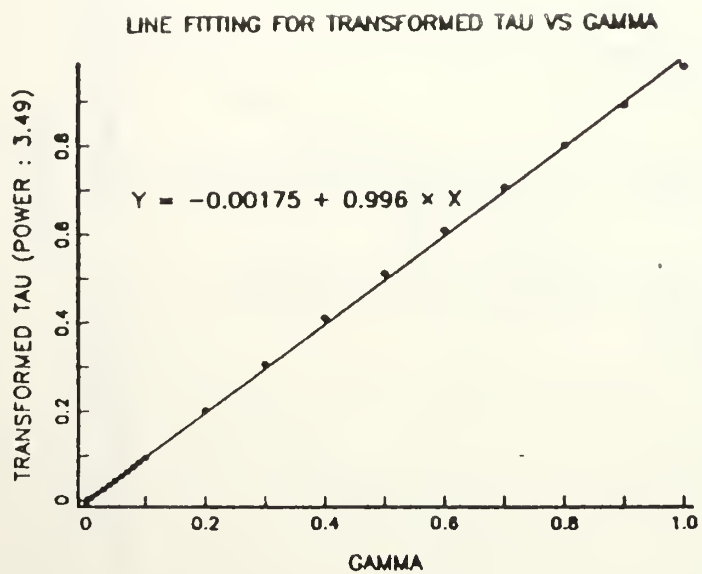


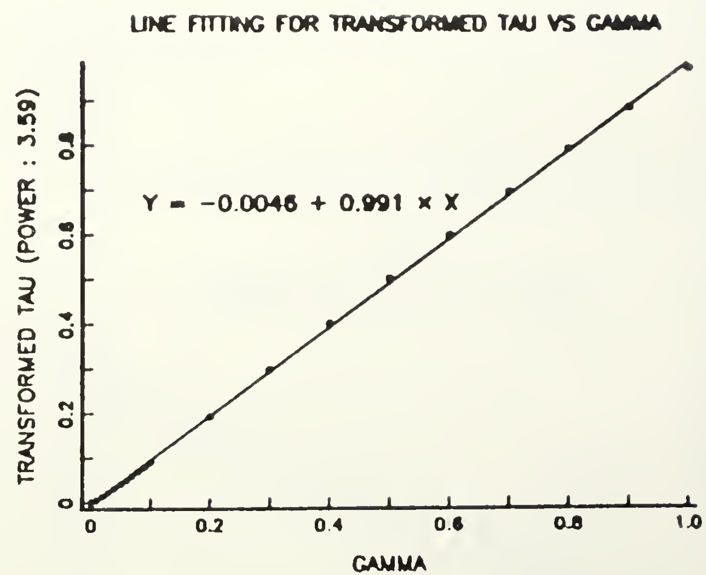
APPENDIX C

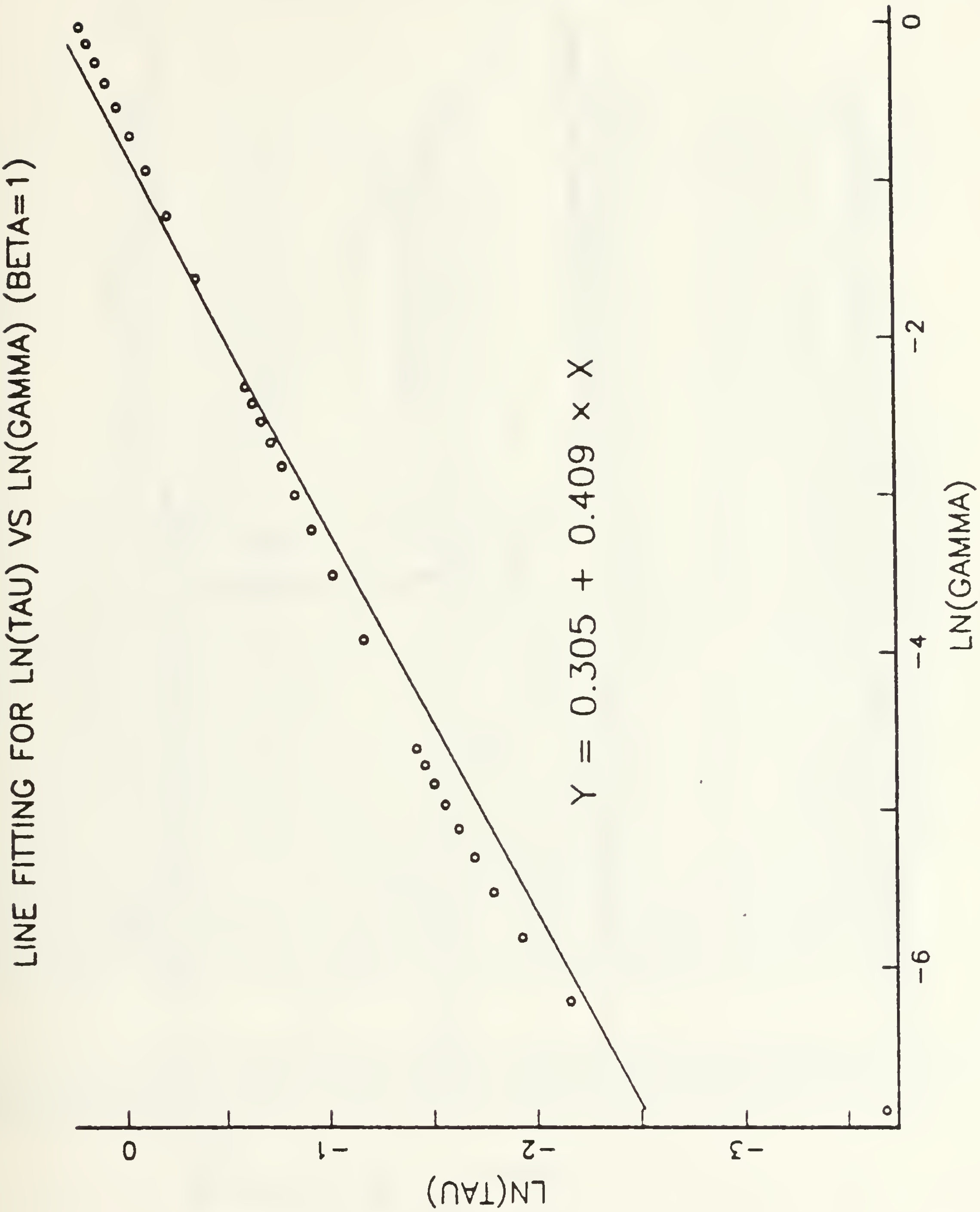
PLOTS OF TAU AND GAMMA TRANSFORMATIONS



LINE FITTING FOR VARIOUS POWER TRANSFORMATIONS

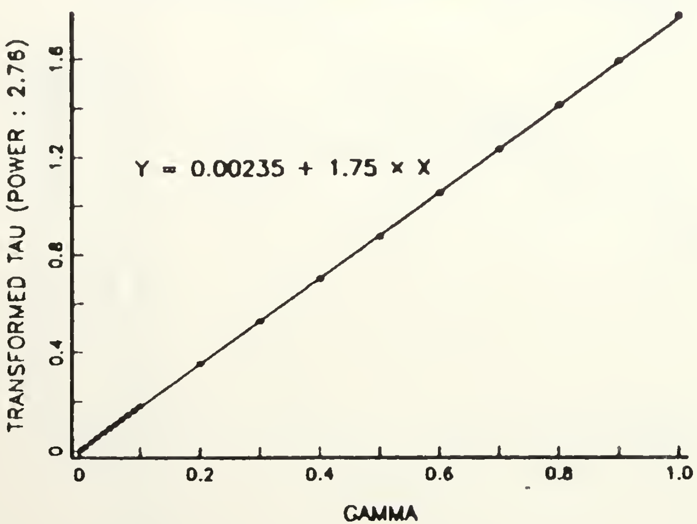




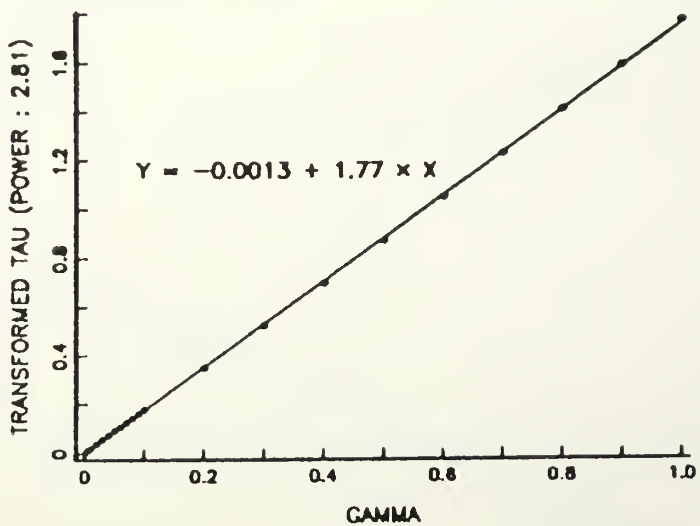

LINE FITTING FOR LN(TAU) VS LN(GAMMA) (BETA=1)
Y = 0.305 + 0.409 × X
0
−2
−4
−6
LN(GAMMA)
0
−1
−2
−3
LN(TAU)

LINE FITTING FOR VARIOUS POWER TRANSFORMATIONS (BETA=1)

LINE FITTING FOR TRANSFORMED TAU VS GAMMA



LINE FITTING FOR TRANSFORMED TAU VS GAMMA



APPENDIX D

COMPUTER SIMULATION PROGRAM

```
***********CALIBRATE
     [0]     CALIBRATE
     [1]    ⍝⍝⍝ DO NOT MOVE OR ERASE; GRAFSTAT FUNCTION HEADER
     [2]    ⍝⍝⍝ GRAFSTAT WILL NOT ADD A LINE TO THIS FUNCTION WITHOUT
     [3]    ⍝⍝⍝ THIS HEADER
     [4]    ⍝***********************************
     [5]    ⍝ INTRODUCE THE SIMULATION
     [6]    ⍝***********************************
     [7]     'WELCOME TO THE CALIBRATION SIMULATION'
     [8]     '    YOU WILL BE TRYING TO DETERMINE THE OPTIMAL TIME FOR A'
     [9]     'SUBMARINE TO COME IN TO PORT FOR INSTRUMENT RE-CALIBRATION.'
     [10]   ⍝***********************************
     [11]   ⍝  DETERMINE THE INPUT PARAMETERS
     [12]   ⍝***********************************
     [13]    '    ENTER A VECTOR OF TIMES (IN ARBITRARY TIME UNITS) AT'
     [14]    'WHICH THE SUBMARINE WILL RETURN TO BASE. FOR EACH OF THESE'
     [15]    'TIMES, THE PROGRAM WILL CALCULATE THE EXPECTED'
     [16]    'EFFECTIVENESS OF THE SUBMARINE. THE TIME THAT CORRESPONDS'
     [17]    'TO MAXIMAL EFFECTIVENESS WILL BE CONSIDERED OPTIMAL.'
     [18]    '(NO FRACTIONS PLEASE.)'
     [19]    T←⎕
     [20]    'HOW MANY UNITS OF TIME DOES IT TAKE TO RECALIBRATE THE'
     [21]    'EQUIPMENT?'
     [22]    TCAL←⎕
     [23]    '    ALTHOUGH THE EFFECTIVENESS OF THE SUBMARINE CHANGES'
     [24]    'CONTINUOUSLY WITH TIME, IN A SIMULATION THE EFFECTIVENESS'
     [25]    'IS ESTIMATED ONLY AT DISCRETE POINTS. THE MORE POINTS YOU'
     [26]    'HAVE, THE SMOOTHER THE EFFECTIVENESS CURVE, BUT THE LONGER'
     [27]    'THE PROGRAM TAKES TO RUN.'
     [28]    '    ENTER HOW MANY TIMES IN A TIME UNIT YOU WANT THE'
     [29]    'EFFECTIVENESS OF THE SUBMARINE MEASURED.'
     [30]    '(NO FRACTIONS PLEASE)'
     [31]    IDELTAT←⎕
     [32]    DELTAT←1÷IDELTAT
     [33]    '    HOW MANY REPLICATIONS OF THE SIMULATION SHOULD BE RUN?'
     [34]    'AGAIN, THE PRECISION OF YOUR ESTIMATES OF THE EFFECTIVENESS'
     [35]    'CURVE GETS BETTER WITH MORE REPLICATIONS, BUT IT WILL TAKE'
     [36]    'LONGER TO RUN THE PROGRAM.'
     [37]    REPS←⎕
     [38]   CALIB:
     [39]    '    EFFECTIVENESS WILL BE MEASURED AS THE PROBABILITY OF'
     [40]    'DAMAGING A TARGET SHIP THAT IS 1000 DISTANCE UNITS AWAY'
     [41]    'FROM YOU.'
     [42]    '    YOUR WEAPON IS A STRAIGHT-RUN TORPEDO WITH A PROXIMITY'
     [43]    'FUSE. YOU WILL FIRE THE TORPEDO ALONG SOME BEARING-- CALL'
     [44]    'THIS ANGLE THETA --AND THE TORPEDO IS SUPPOSED TO EXPLODE'
     [45]    'AT THE POINT NEAREST TO THE TARGET.'
     [46]    '    UNFORTUNATELY, YOUR EQUIPMENT TO LOCATE THE TARGET WILL'
     [47]    'DEVELOP CALIBRATION PROBLEMS WITH TIME. DO YOU WANT THE'
     [48]    'CALIBRATION PROBLEM TO:'
     [49]    '        GET DETERMINISTICALLY WORSE WITH TIME    (ENTER 0)'
```

```
**********CALIBRATE      cont'd
     [50]     '        GET RANDOMLY WORSE WITH TIME                 (ENTER 1)'
     [51]     '        FLUCTUATE RANDOMLY WITH TIME                 (ENTER 2)'
     [52]     '        BE RANDOM WITH TIME (GAMMA DIST.)            (ENTER 3)'
     [53]     '(IN THE THIRD CHOICE, CALIBRATION CAN IMPROVE OR WORSEN'
     [54]     'WITH TIME)'
     [55]     RAND←⎕
    →[56]     →((RAND=0),(RAND=1),(RAND=2),(RAND=3))/CONS,RANDOM,FLUC,GAMMA
     [57]    CONS:
     [58]     'HOW MANY DISTANCE UNITS (FRACTIONS ARE OKAY) WILL'
     [59]     'CALIBRATION DRIFT FOR EVERY TIME UNIT?'
     [60]     RATE←⎕
     [61]     AP←RATE
     [62]     AERR←'CONSTANT'
    →[63]     →DELIVER
     [64]    RANDOM:
     [65]     '     ON DAY T, THE DRIFT WILL BE MISMEASURED BY AN AMOUNT'
     [66]     'T×NORMAL(0,SIGMA). ALTHOUGH THE MEAN ERROR IS ZERO, THE'
     [67]     'VARIANCE OF THE ERROR WILL INCREASE AS SIGMA TIMES TIME,'
     [68]     'SQUARED. NOTICE THAT THE RANDOM MULTIPLIER WILL BE CONSTANT'
     [69]     'IN EACH REPLICATION OF THE SIMULATION.'
     [70]     '     HOW LARGE, IN DISTANCE UNITS SHOULD SIGMA BE?'
     [71]     UERR←⎕
     [72]     AP←UERR
     [73]     AERR←'RANDOM'
    →[74]     →DELIVER
     [75]    FLUC:
     [76]     '     ON EACH DAY, THE DRIFT WILL BE MISMEASURED BY AN'
     [77]     'ADDITIONAL DRIFT ERROR. THIS ERROR TERM WILL BE RANDOM AND'
     [78]     'COME FROM A NORMAL(0,SIGMA) DISTRIBUTION, WHERE SIGMA IS'
     [79]     'EXPRESSED IN DISTANCE UNITS AND REPRESENTS THE STANDARD'
     [80]     'DEVIATION OF THE ERROR DISTRIBUTION. NOTICE THAT THE'
     [81]     'EXPECTED ERROR IS ALWAYS ZERO, ALTHOUGH THE VARIANCE GROWS'
     [82]     'PROPORTIONALLY WITH TIME.'
     [83]     '     HOW LARGE, IN DISTANCE UNITS DO YOU WANT SIGMA TO BE?'
     [84]     NERR←⎕
     [85]     AP←NERR
     [86]     AERR←'NORMAL'
    →[87]     →DELIVER
     [88]    GAMMA:
     [89]     '     CALIBRATION DRIFT FOR EVERY TIME UNIT WILL BE GAMMA'
     [90]     'RANDOM VARIABLE WITH SHAPE PARAMETERS LAMBDA AND BETA.'
     [91]     'WHAT PARAMETERS DO YOU WANT TO USE TO GENERATE GAMMA RANDOM'
     [92]     'DRIFT?'
     [93]     'FOR LAMBDA'
     [94]     LAMBDA←⎕
     [95]     'FOR BETA'
     [96]     BETA←⎕
     [97]     AB←BETA,LAMBDA
     [98]     RATE←AB GAMMACK 1
     [99]     AP←RATE
```

```
**********CALIBRATE     cont'd
   [100]   AERR←'GAMMA'
   [101]  DELIVER:
   [102]   '    YOUR TORPEDO WILL BE AIMED AT A POINT DETERMINED BY YOUR'
   [103]   'CALIBRATION ERROR. IN ADDITION, THE TORPEDO MAY NOT EXPLODE'
   [104]   'AT PRECISELY THE POINT ON THAT BEARING THAT IS CLOSEST TO'
   [105]   'THE TARGET, I.E. THE PROXIMITY FUSE IS NOT PERFECTLY'
   [106]   'ACCURATE. THE ERROR BETWEEN THE CLOSEST POINT AND THE'
   [107]   'EXPLOSION POINT CAN COME FROM ANY OF THE FOLLOWING'
   [108]   'DISTRIBUTIONS:'
   [109]   '              NORMAL     (ENTER 0)'
   [110]   '              UNIFORM    (ENTER 1)'
   [111]   '              NO ERROR   (ENTER 2)'
   [112]   DIST←⎕
  →[113]   →((DIST=0),(DIST=1),(DIST=2))/NORMAL,UNIF,NOERR
   [114]  NORMAL:
   [115]   'WHAT VARIANCE DO YOU WANT TO USE IN THE X-DIRECTION?'
   [116]   XVAR←⎕
   [117]   'WHAT VARIANCE DO YOU WANT TO USE IN THE Y-DIRECTION?'
   [118]   YVAR←⎕
   [119]   FP1←XVAR
   [120]   FP2←YVAR
   [121]   FERR←'NORMAL'
  →[122]   →DAMAGE
   [123]  UNIF:
   [124]   '    THE ERROR IN THE X-DIRECTION WILL BE UNIFORM[¯X,X].'
   [125]   'HOW MANY DISTANCE UNITS SHOULD X BE?'
   [126]   XUNI←⎕
   [127]   '    THE ERROR IN THE Y-DIRECTION WILL BE UNIFORM[¯Y,Y].'
   [128]   'HOW MANY DISTANCE UNITS SHOULD Y BE?'
   [129]   YUNI←⎕
   [130]   FP1←XUNI
   [131]   FP2←YUNI
   [132]   FERR←'UNIFORM'
  →[133]   →DAMAGE
   [134]  NOERR:
   [135]   FP1←' '
   [136]   FP2←' '
   [137]   FERR←'NONE'
   [138]  DAMAGE:
   [139]   '    THE TARGET WILL BE DAMAGED WITH PROBABILITY CALCULATED'
   [140]   'ACCORDING TO SOME FUNCTION.  WHAT FUNCTION DO YOU WANT TO'
   [141]   'USE?'
   [142]   '             GAUSS DIFFUSE DAMAGE     (ENTER 0)'
   [143]   '                  COOKIE CUTTER       (ENTER 1)'
   [144]   '                    TRAPEZOIDAL       (ENTER 2)'
   [145]   DAM←⎕
  →[146]   →((DAM=0),(DAM=1),(DAM=2))/NORD,COOKD,TRAPD
   [147]  NORD:
   [148]   'WHAT PARAMETER DO YOU WANT TO USE IN THE GAUSS MODEL?'
   [149]   ALPHA←⎕
```

```
**********CALIBRATE     cont'd
    [150]   DP1←ALPHA
    [151]   DP2←' '
    [152]   DPROB←'GAUSS'
   →[153]   →STARTS
    [154]  COOKD:
    [155]   '    THE TORPEDO WILL DESTROY THE TARGET IF IT IS WITHIN A'
    [156]   'CERTAIN RADIUS, AND IT WILL MISS IF IT IS OUTSIDE THIS'
    [157]   'RADIUS.'
    [158]   '    HOW MANY DISTANCE UNITS SHOULD THIS RADIUS BE?'
    [159]   COOKRAD←⎕
    [160]   DP1←COOKRAD
    [161]   DP2←' '
    [162]   DPROB←'COOKIE'
   →[163]   →STARTS
    [164]  TRAPD:
    [165]   '    THE TRAPEZOIDAL DAMAGE FUNCTION HAS A CENTRAL, CIRCULAR'
    [166]   'PLATEAU OF RADIUS R1. IF THE TARGET IS WITHIN R1 DISTANCE'
    [167]   'UNITS OF THE EXPLODING TORPEDO, THEN THE TARGET IS DAMAGED'
    [168]   'WITH PROBABILITY ONE. IN ADDITION, THE FUNCTION HAS AN'
    [169]   'OUTER CIRCULAR RIM, OF RADIUS R2, BEYOND WHICH THE'
    [170]   'PROBABILITY OF DAMAGING THE SHIP IS ZERO. BETWEEN THE TWO'
    [171]   'RADIUS, THE DAMAGE PROBABILITY GOES DOWN LINEARLY.'
    [172]   '    HOW MANY DISTANCE UNITS SHOULD THE INNER RADIUS BE?'
    [173]   R1←⎕
    [174]   '    KEEPING IN MIND THAT THE OUTER RADIUS MUST BE LARGER'
    [175]   'THAN THE INNER ONE JUST ENTERED, HOW MANY DISTANCE UNITS'
    [176]   'SHOULD THE OUTER RADIUS BE?'
    [177]   R2←⎕
    [178]   DP1←R1
    [179]   DP2←R2
    [180]   DPROB←'TRAPEZOID'
   →[181]   →((R2>R1),(R2≤R1))/STARTS,TRAPD
    [182]  ⍝*****************************************
    [183]  ⍝   START THE SIMUALTION CALCULATIONS
    [184]  ⍝*****************************************
    [185]  STARTS:
    [186]  ⍝
    [187]  ⍝  INITIALIZE THE OUTPUT ARRAYS
    [188]  ⍝ 'EFF'  WILL CONTAIN THE ESTIMATED EFECTIVENESS AT EACH TIME
    [189]  ⍝        INCREMENT DELTAT OUT TO THE MAXIMUM TIME IN THE
    [190]  ⍝        INPUT TIME VECTOR T
    [191]  ⍝ 'TEFF'  WILL BE THE EFFECTIVENESS FOR A GIVEN REPLICATION;
    [192]  ⍝         IT IS REALLY JUST TEMPORARY STORAGE FOR 'EFF'
    [193]  ⍝ 'TIMES'  WILL BE THE TIMES AT WHICH 'EFF' WILL BE EVALUATED
    [194]  ⍝ 'STDEFF'  WILL CONTAIN THE STANDARD DEVIATION OF THE
    [195]  ⍝           EFFECTIVENESS
    [196]  ⍝ 'AVGEFF'  WILL CONTAIN THE AVERAGE EFFECTIVENESS IF THE SHIP
    [197]  ⍝           RETURNS AT THE VARIOUS TIMES IN THE VECTOR 'T'
    [198]  ⍝
    [199]   NOPT←⍴T
```

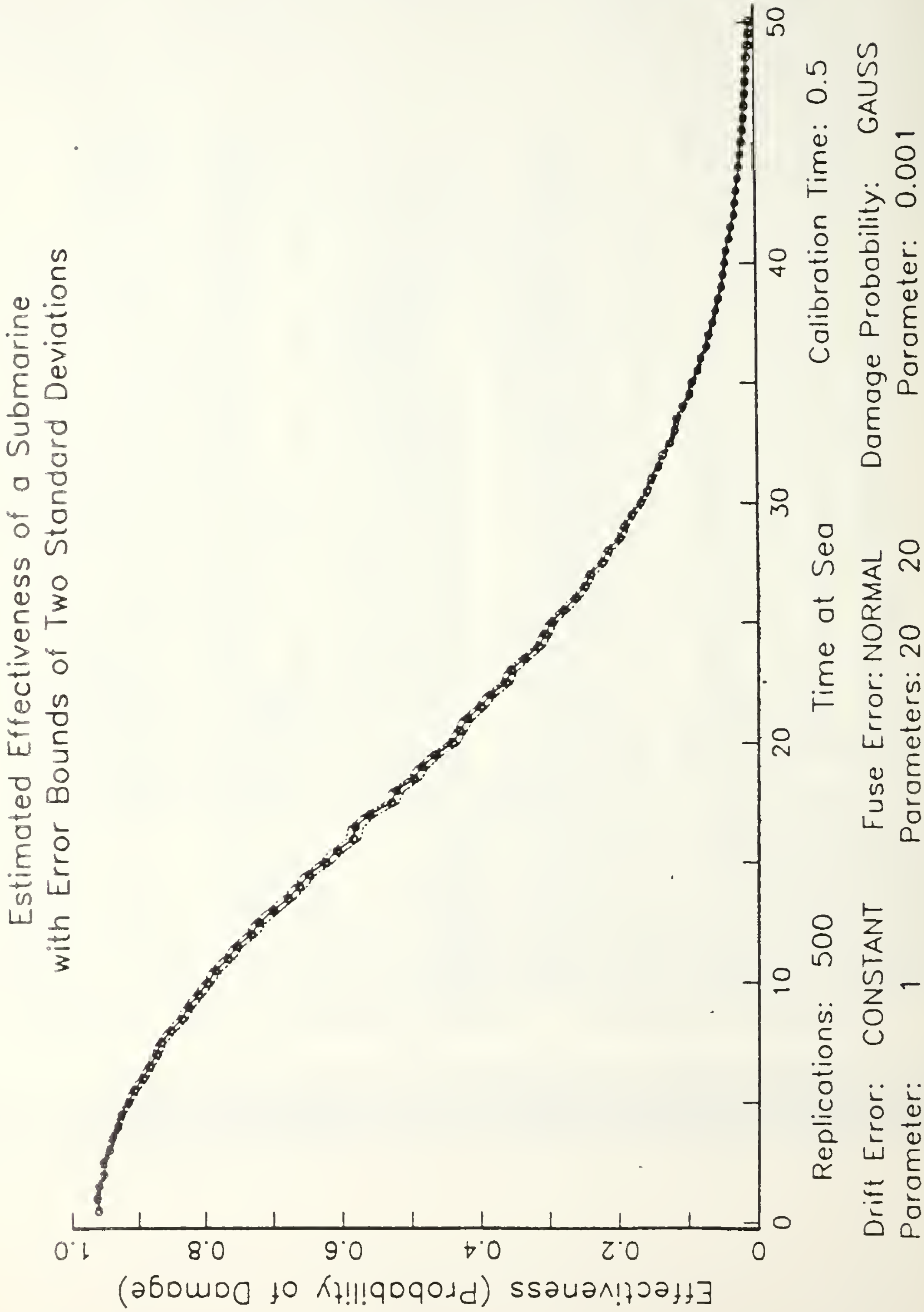
```
**********CALIBRATE     cont'd
    [200]   T←T[⍋T]
    [201]   EFF←(T[NOPT]÷DELTAT)⍴0
    [202]   NEFF←⍴EFF
    [203]   STDEFF←EFF
    [204]   AVGEFF←NOPT⍴0
    [205]   NREPS←0
    [206]   TIMES←DELTAT×(⍳NEFF)
    [207]  ⍝
    [208]  ⍝  LOOP THROUGH THE ALGORITHM 'REPS' TIMES
    [209]  ⍝
    [210]  LOOP:NREPS←NREPS+1
    [211]  ⍝ GET THE CALIBRATION ERRORS IN THE X AND Y DIRECTIONS
   →[212]   →((RAND=0),(RAND=1),(RAND=2),(RAND=3))/CCON,CRAN,CNOR,CGAM
    [213]  CCON:
    [214]   XERR←RATE×TIMES
    [215]   YERR←XERR
   →[216]   →ORIENT
    [217]  CRAN:
    [218]   MAX←2147483646
    [219]   XERR←(?(2⍴MAX))÷MAX
    [220]   XERR←((-2×(⍟(XERR[1])))*0.5)×(1○(○(2×(XERR[2]))))
    [221]   XERR←XERR×UERR×TIMES
    [222]   YERR←XERR
   →[223]   →ORIENT
    [224]  CNOR:
    [225]   MAX←2147483646
    [226]   XERR←(?((T[NOPT])⍴MAX))÷MAX
    [227]   TEMPU←(?((T[NOPT])⍴MAX))÷MAX
    [228]   TEMPU←(-2×(⍟TEMPU))*0.5
    [229]   XERR←TEMPU×(1○(○(2×XERR)))
    [230]   XERR←XERR×NERR×DELTAT
    [231]   XERR←,⍉((IDELTAT,(T[NOPT]))⍴XERR)
    [232]   XERR←+\XERR
    [233]   YERR←XERR
   →[234]   →ORIENT
    [235]  CGAM:
    [236]   XERR←RATE×TIMES
    [237]   YERR←XERR
    [238]  ORIENT:
    [239]  ⍝ REORIENT PERSPECTIVE, I.E. ROTATE YOUR VIEW OF THE TARGET
    [240]  ⍝ BY 'THETA' SO THAT YOU FACE THE POINT YOU FIRE AT, NOT THE
    [241]  ⍝ TARGET
    [242]   RADIUS←((XERR×XERR)+((YERR+1000)*2))*0.5
    [243]   XMEAN←NEFF⍴0
    [244]   YMEAN←RADIUS
    [245]   THETA←¯2○(XERR÷RADIUS)
    [246]   TARGX←1000×(2○((○1)-THETA))
    [247]   TARGY←1000×(1○((○1)-THETA))
    [248]  ⍝  GENERATE RANDOM ERRORS FOR THE TORPEDO RUNS
   →[249]   →((DIST=0),(DIST=1),(DIST=2))/NORERR,UNIERR,NOERROR
```

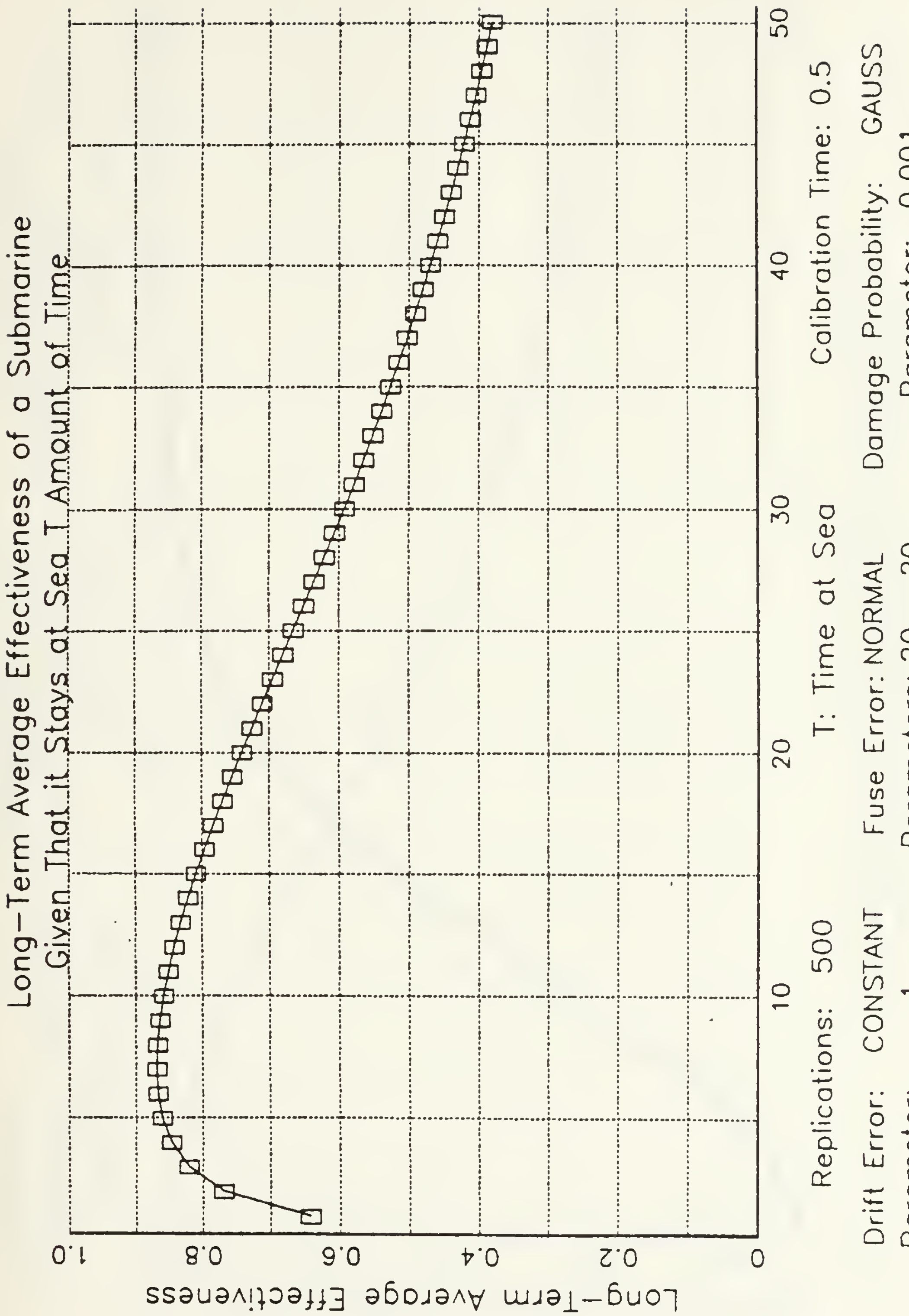
```
**********CALIBRATE     cont'd
 NORERR:
 ⍝  GENERATE STANDARD NORMAL(0,1) ERRORS AND ADJUST THE
 ⍝  VARIANCES
  MAX←2147483646
  TEMPU←(?(NEFF⍴MAX))÷MAX
  TORPX←(?(NEFF⍴MAX))÷MAX
  TEMPU←(-2×(⍟TEMPU))*0.5
  TORPY←TEMPU×(1○(○(2×TORPX)))
  TORPX←TEMPU×(2○(○(2×TORPX)))
  TORPY←TORPY×(YVAR*0.5)
  TORPX←TORPX×(XVAR*0.5)
  →ADDMEAN
 UNIERR:
 ⍝  GENERATE UNIFORM(0,1) ERRORS AND ADJUST LOCATION AND SCALE
  MAX←2147483646
  TORPX←(?(NEFF⍴MAX))÷MAX
  TORPY←(?(NEFF⍴MAX))÷MAX
  TORPX←(TORPX-0.5)×2×XUNI
  TORPY←(TORPY-0.5)×2×YUNI
  →ADDMEAN
 NOERROR:
  TORPX←NEFF⍴0
  TORPY←TORPX
 ADDMEAN:
 ⍝  ADJUST THE FINAL LOCATION OF THE TORPEDO TO REFLECT THE
 ⍝  ERROR PLUS THE POORLY CALIBRATED LOCATION OF THE TARGET
  TORPX←TORPX+XMEAN
  TORPY←TORPY+YMEAN
 ⍝  CALCULATE THE EFFECTIVENESS, I.E. THE PROBABILITY OF
 ⍝  SINKING THE TARGET, GIVEN THAT THE TARGET IS AT THE
 ⍝  (REORIENTED) POINT (TARGX,TARGY) AND THE TORPEDO IS AT
 ⍝  THE POINT (TORPX,TORPY)
  →((DAM=0),(DAM=1),(DAM=2))/NOREFF,COOKEFF,TRAPEFF
 NOREFF:
  TEFF←((TARGX-TORPX)*2)+((TARGY-TORPY)*2)
  TEFF←*(-(ALPHA×TEFF))
  →ACCUM
 COOKEFF:
  TEFF←(((TARGX-TORPX)*2)+((TARGY-TORPY)*2))*0.5
  TEFF←TEFF≤COOKRAD
  →ACCUM
 TRAPEFF:
  TEFF←(((TARGX-TORPX)*2)+((TARGY-TORPY)*2))*0.5
  BET←((TEFF>R1)∧(TEFF≤R2))
  BET←((1÷(R2-R1))×(R2-TEFF))×BET
  TEFF←BET+(TEFF≤R1)
 ACCUM:
 ⍝  ADD THE EFFECTIVENESS FOR THIS REPLICATION TO 'EFF';
 ⍝  SIMILARLY UPDATE THE STANDARD DEVIATION OF THE
 ⍝  EFFECTIVENESS 'STDEFF'
```

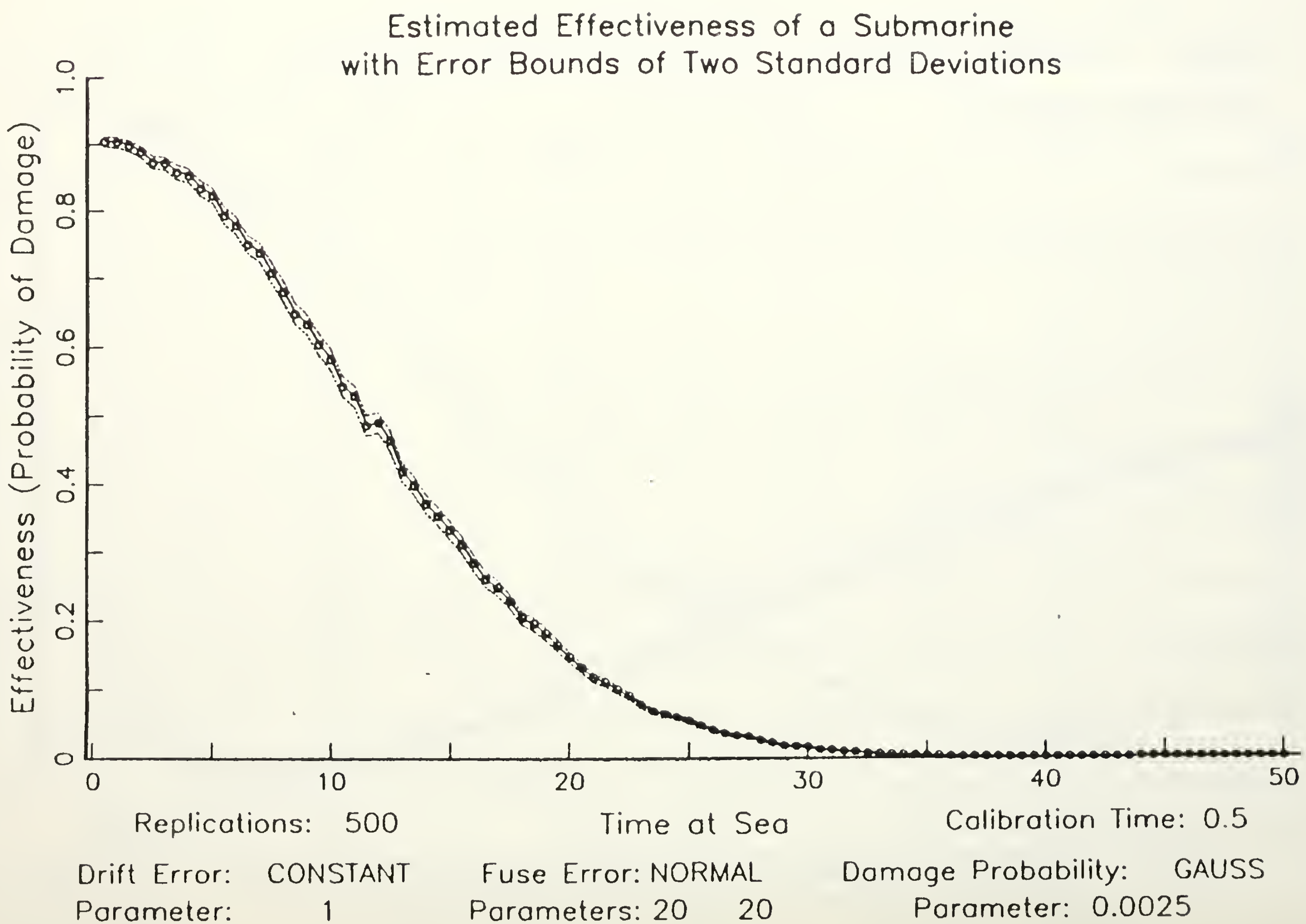
```
**********CALIBRATE     cont'd
     [300]   EFF←EFF+TEFF
     [301]   STDEFF←STDEFF+(TEFF×TEFF)
    →[302]   →(NREPS<REPS)/LOOP
     [303]   EFF←EFF÷REPS
     [304]   STDEFF←(STDEFF-(REPS×EFF×EFF))÷(REPS×(REPS-1))
     [305]   STDEFF←STDEFF×(STDEFF≥0)
     [306]   STDEFF←STDEFF*0.5
     [307]  ⍝  CALCULATE THE AVERAGE EFFECTIVENESS FOR THE POSSIBLE TIMES
     [308]  ⍝  THAT THE SHIP IS OUT
     [309]   NTIMES←0
     [310]  LOOP2:NTIMES←NTIMES+1
     [311]   NPTS←T[NTIMES]÷DELTAT
     [312]   AVGEFF[NTIMES]←(+/(EFF[⍳NPTS]))×DELTAT
     [313]   AVGEFF[NTIMES]←AVGEFF[NTIMES]÷(T[NTIMES]+TCAL)
    →[314]   →(NTIMES<NOPT)/LOOP2
     [315]  ⍝
     [316]  ⍝  CHECK TO SEE IF THE USER HAS GRAPHICS AVAILABLE AND
     [317]  ⍝  WANTS TO USE THEM
     [318]  ⍝
     [319]   '  DO YOU HAVE GRAFSTAT LOADED AND WISH TO SEE PLOTS OF'
     [320]   'THE EFFECTIVENESS CURVE WITH TIME, AND'
     [321]   'THE AVERAGE LONG-RUN EFFECTIVENESS FOR THE TIMING OPTIONS'
     [322]   'THAT YOU INPUT?'
     [323]   '            NO=0'
     [324]   '           YES=1'
     [325]   GRAPHICS←⎕
    →[326]   →((GRAPHICS=0),(GRAPHICS=1))/MESSAGE,PLOT
     [327]  MESSAGE:
     [328]   'YOUR OUTPUT CAN BE FOUND IN THE FOLLOWING VECTORS:'
     [329]   '       EFF    CONTAINS THE ESTIMATED EFFECTIVENESS, AT'
     [330]   '              THE TIME INTERVALS YOU SPECIFIED.'
     [331]   '              EFFECTIVENESS IS SIMPLY THE PROBABILITY OF'
     [332]   '              THE TORPEDO DESTROYING ITS TARGET'
     [333]   ' '
     [334]   '    STDEFF    CONTAINS THE STANDARD DEVIATIONS OF THE'
     [335]   '              ESTIMATES IN EFF ABOVE'
     [336]   ' '
     [337]   '    AVGEFF    CONTAINS THE LONG-TERM AVERAGE EFFECTIVENESS'
     [338]   '              OF THE SUBMARINE IF IT RETURNS AFTER THE'
     [339]   '              NUMBER OF TIME UNITS INPUT BY YOU AND'
     [340]   '              SPECIFIED IN THE VECTOR T BELOW'
     [341]   ' '
     [342]   '         T    YOUR INPUT VECTOR OF TIMES WHEN THE SUBMARINE'
     [343]   '              SHOULD BE BROUGHT BACK FOR EQUIPMENT'
     [344]   '              RECALIBRATION.  THESE VALUES ARE NOW ORDERED,'
     [345]   '              IF THEY WERE NOT BEFORE.'
```

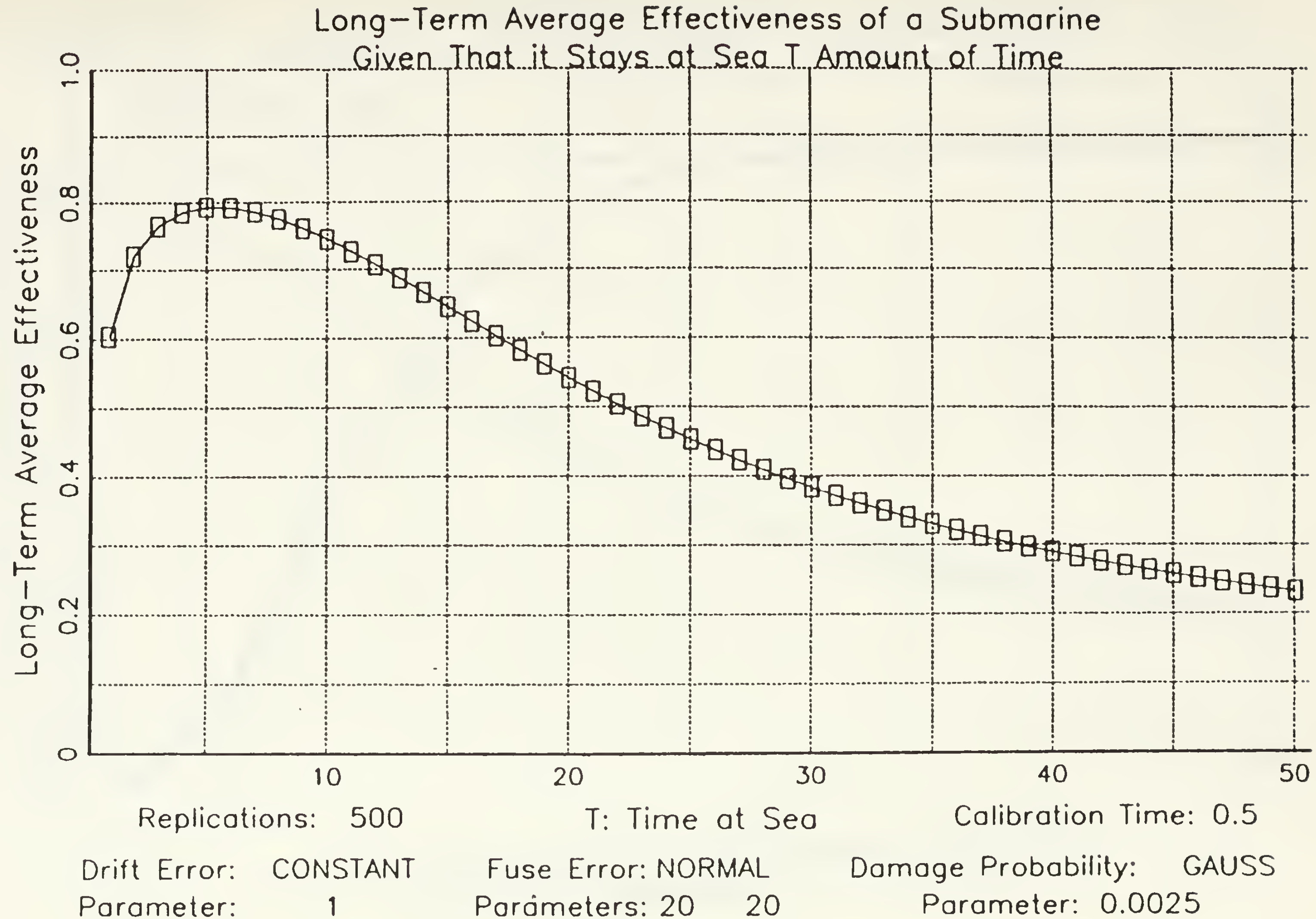
APPENDIX E

PLOTS OF SIMULATION RESULTS

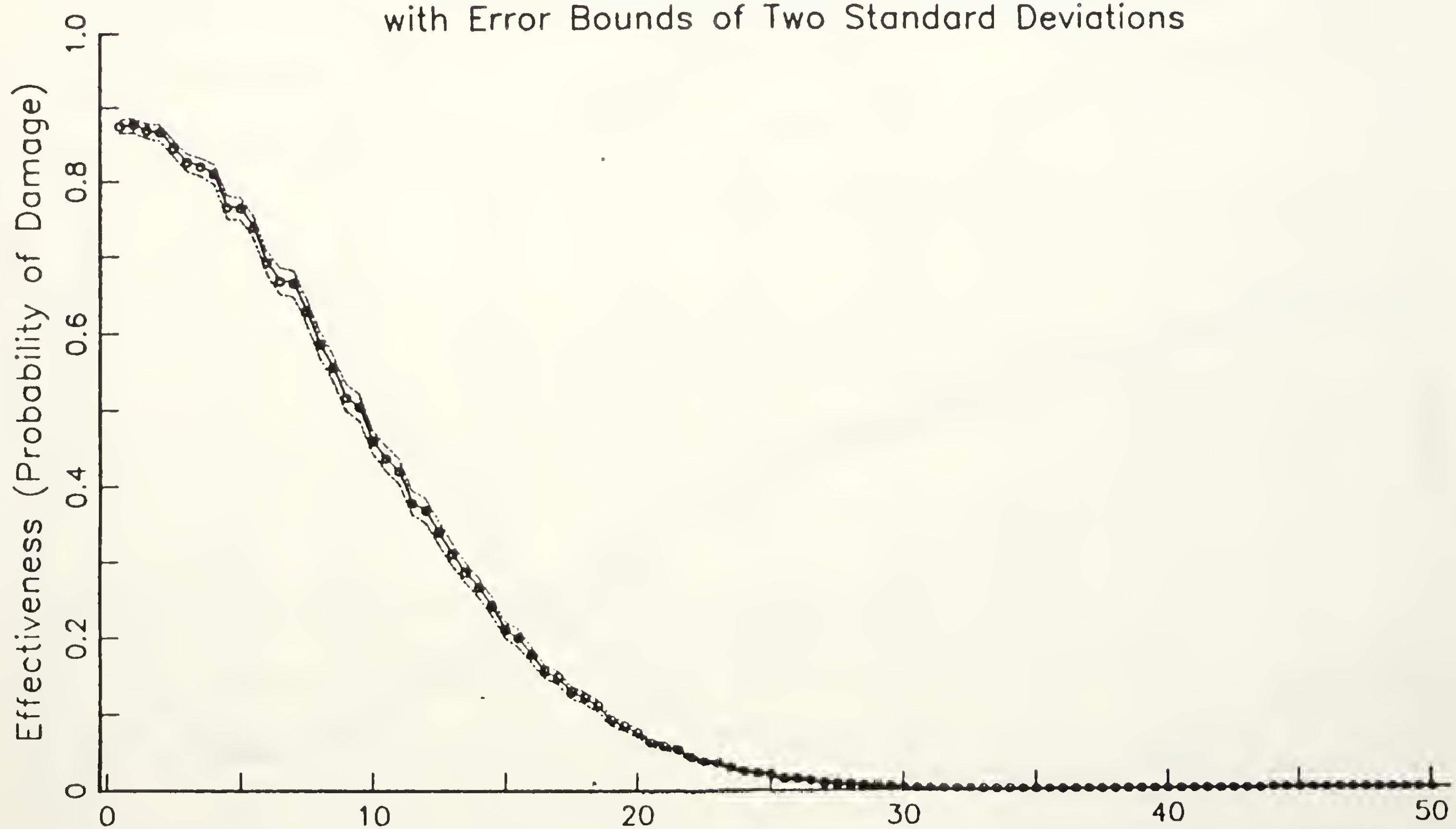



Long-Term Average Effectiveness of a Submarine
Given That it Stays at Sea T Amount of Time
Long-Term Average Effectiveness
0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
T: Time at Sea
Replications: 500
Calibration Time: 0.5
Drift Error: CONSTANT
Parameter: 1
Fuse Error: NORMAL
Parameters: 20 20
Damage Probability: GAUSS
Parameter: 0.001


Estimated Effectiveness of a Submarine
with Error Bounds of Two Standard Deviations
Effectiveness (Probability of Damage)
0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
Time at Sea
Replications: 500
Calibration Time: 0.5
Drift Error: CONSTANT
Parameter: 1
Fuse Error: NORMAL
Parameters: 20 20
Damage Probability: GAUSS
Parameter: 0.0025


Long-Term Average Effectiveness of a Submarine
Given That it Stays at Sea T Amount of Time
Long-Term Average Effectiveness
0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
T: Time at Sea
Replications: 500
Calibration Time: 0.5
Drift Error: CONSTANT
Parameter: 1
Fuse Error: NORMAL
Parameters: 20 20
Damage Probability: GAUSS
Parameter: 0.0025

Estimated Effectiveness of a Submarine
with Error Bounds of Two Standard Deviations

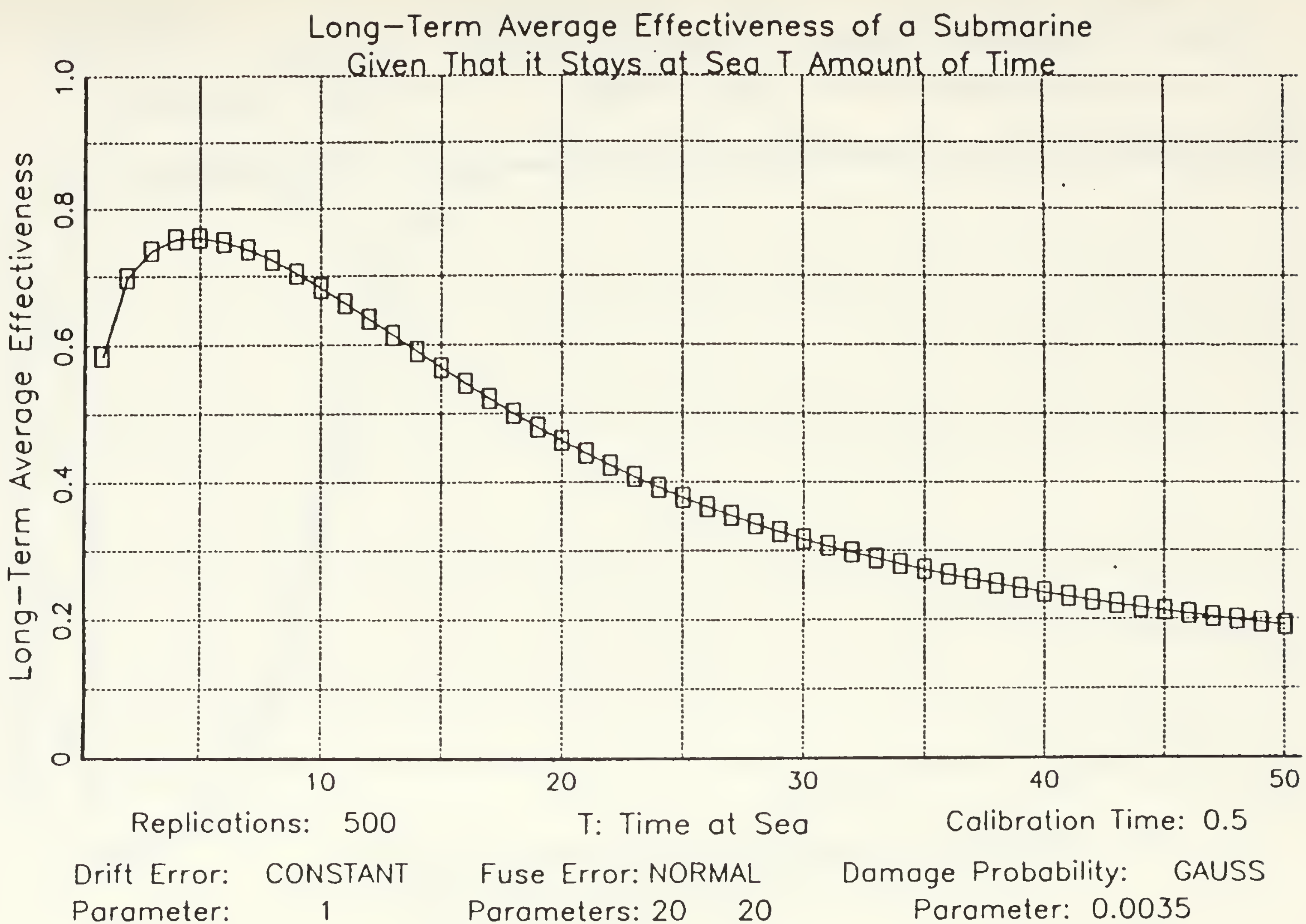


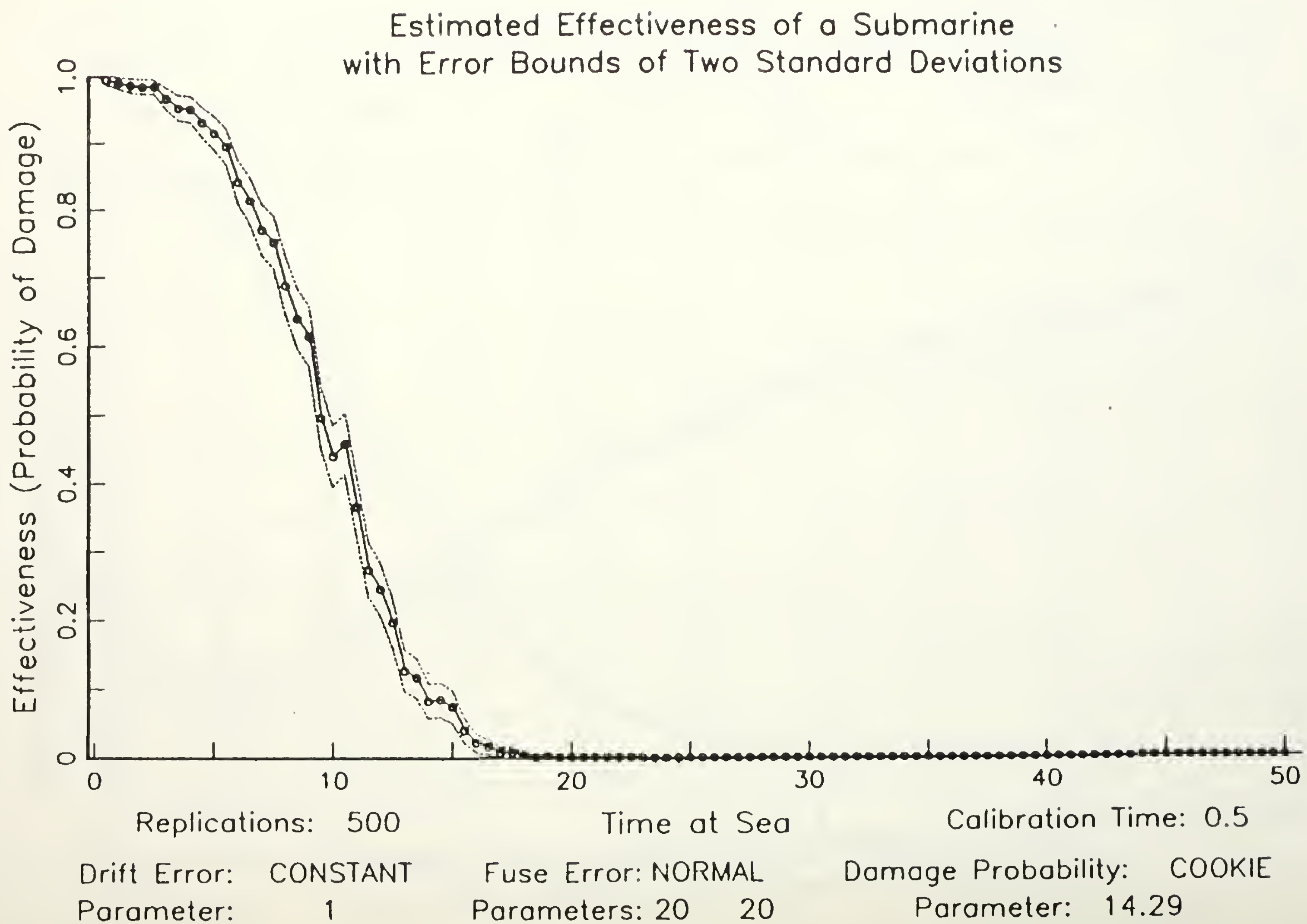
Replications: 500
Calibration Time: 0.5

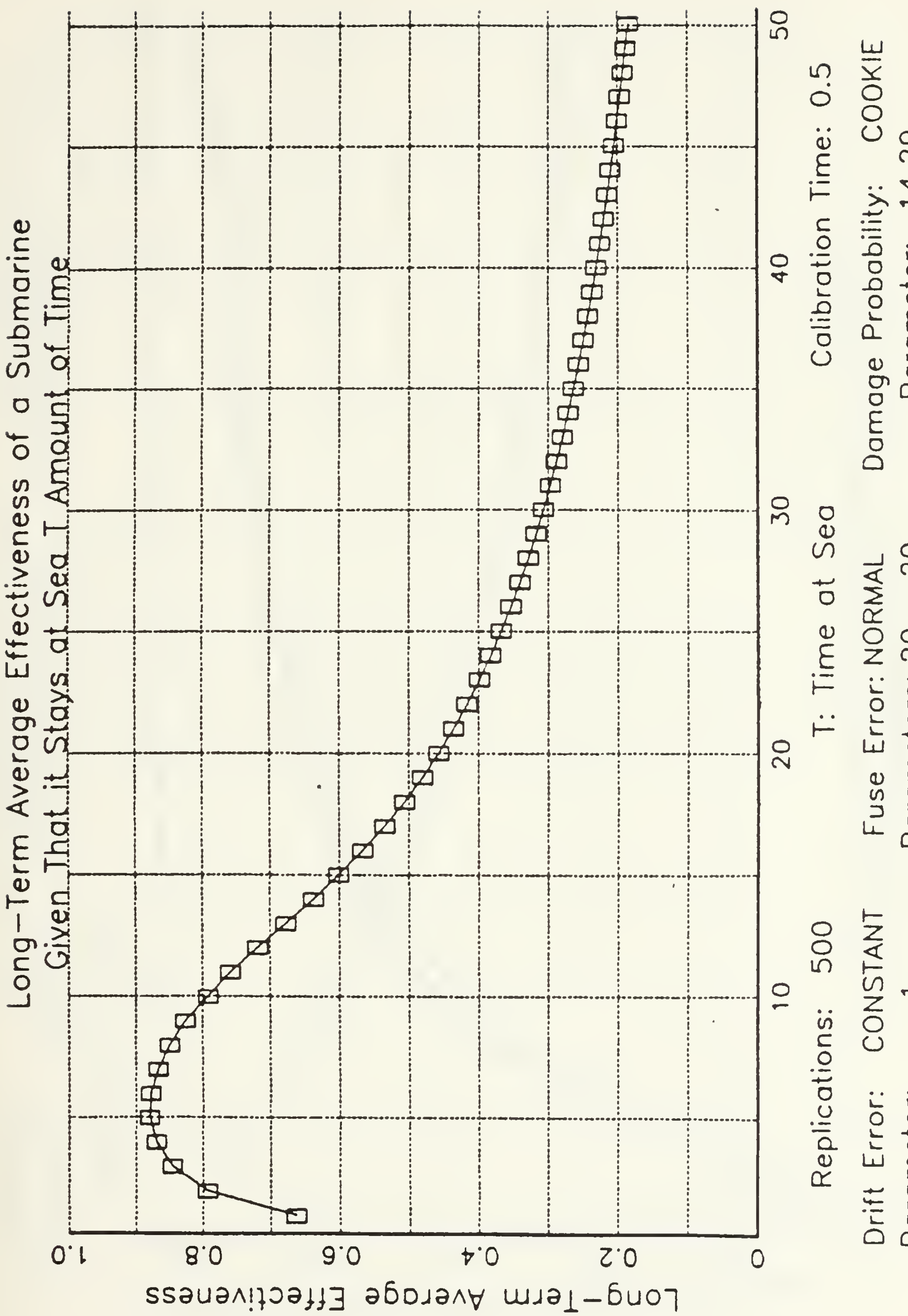
Drift Error: CONSTANT
Parameter: 1

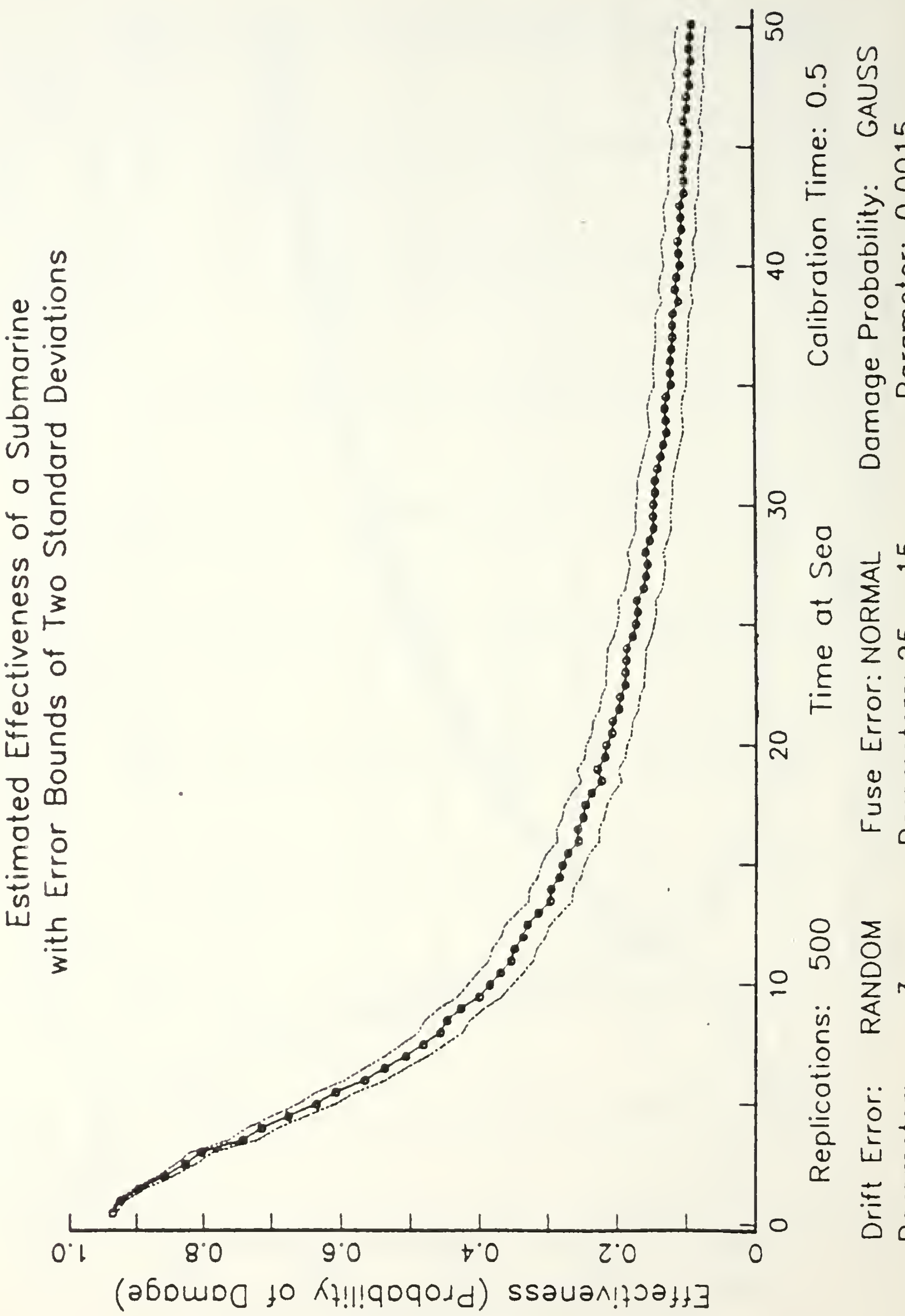
Fuse Error: NORMAL
Parameters: 20 20

Damage Probability: GAUSS
Parameter: 0.0035

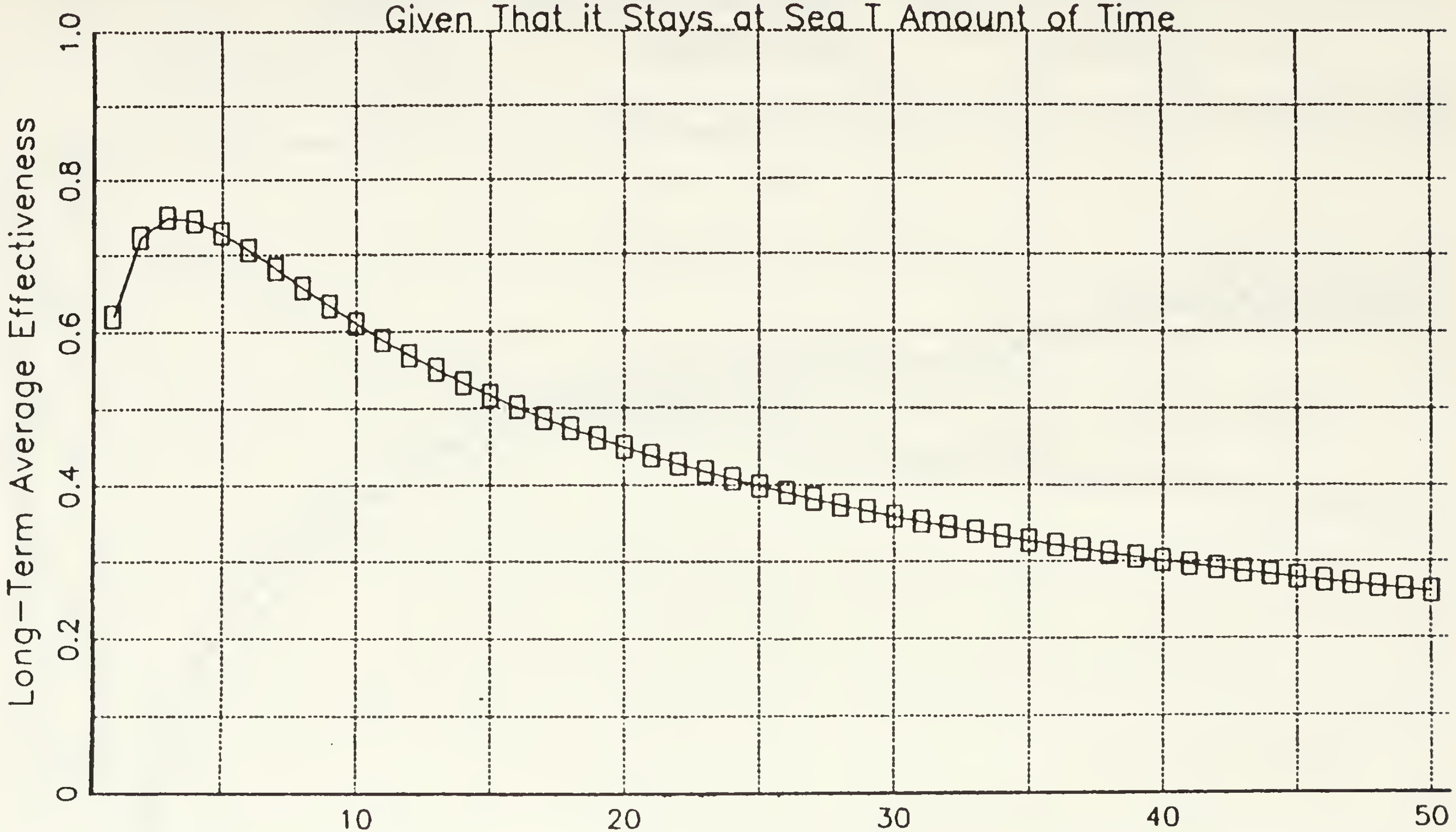

Long-Term Average Effectiveness of a Submarine
Given That it Stays at Sea T Amount of Time
Long-Term Average Effectiveness
0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
Replications: 500
T: Time at Sea
Calibration Time: 0.5
Drift Error: CONSTANT
Parameter: 1
Fuse Error: NORMAL
Parameters: 20 20
Damage Probability: GAUSS
Parameter: 0.0035


Estimated Effectiveness of a Submarine
with Error Bounds of Two Standard Deviations
Effectiveness (Probability of Damage)
0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
Replications: 500
Time at Sea
Calibration Time: 0.5
Drift Error: CONSTANT
Parameter: 1
Fuse Error: NORMAL
Parameters: 20 20
Damage Probability: COOKIE
Parameter: 14.29


Long-Term Average Effectiveness of a Submarine
Given That it Stays at Sea T Amount of Time
Long-Term Average Effectiveness
0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
T: Time at Sea
Replications: 500
Calibration Time: 0.5
Drift Error: CONSTANT
Fuse Error: NORMAL
Damage Probability: COOKIE
Parameter: 1
Parameters: 20 20
Parameter: 14.29

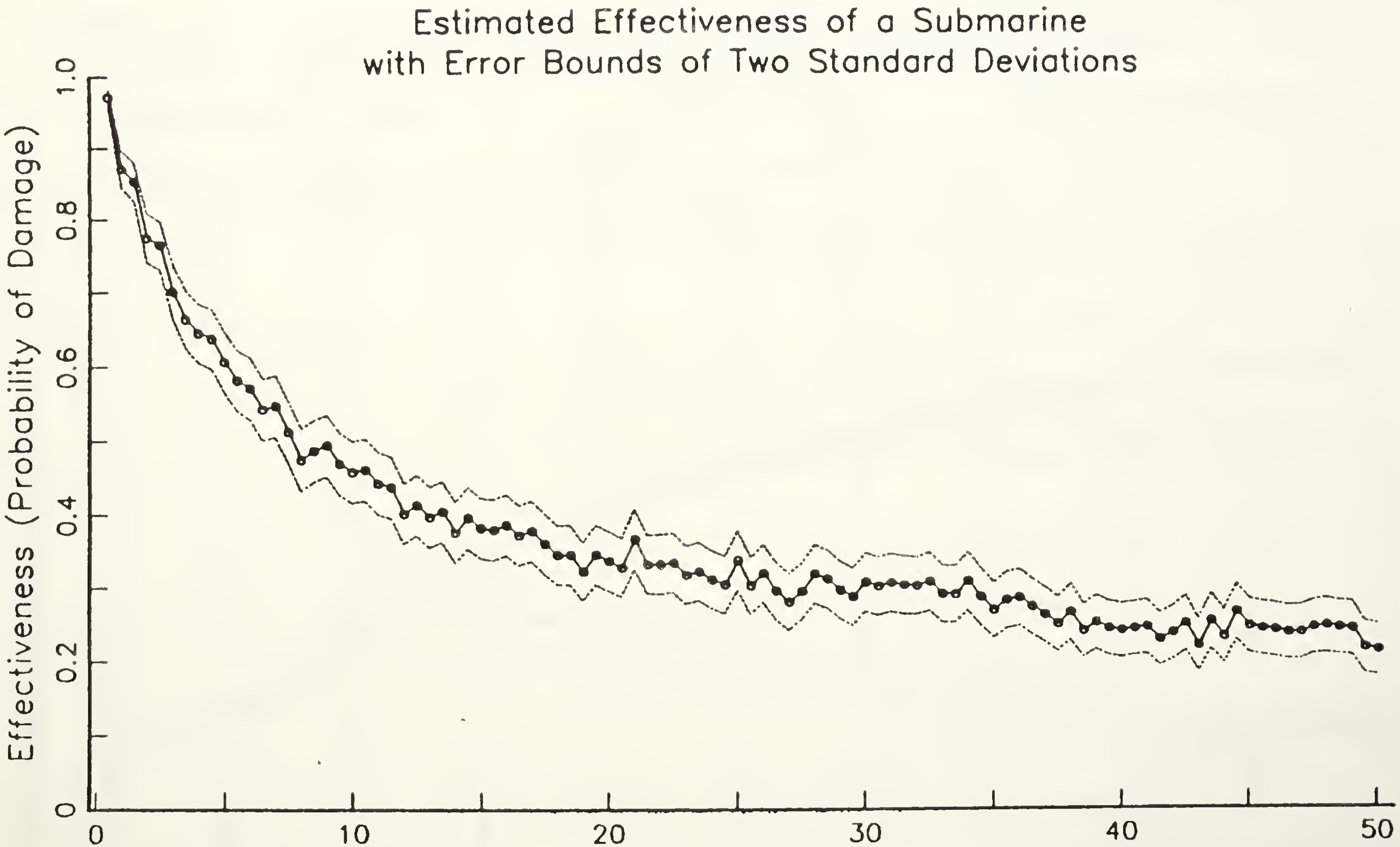

Estimated Effectiveness of a Submarine
with Error Bounds of Two Standard Deviations
Effectiveness (Probability of Damage)
0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
Time at Sea
Replications: 500
Calibration Time: 0.5
Drift Error: RANDOM
Fuse Error: NORMAL
Damage Probability: GAUSS
Parameter: 3
Parameters: 25 15
Parameter: 0.0015

Long-Term Average Effectiveness of a Submarine
Given That it Stays at Sea T Amount of Time



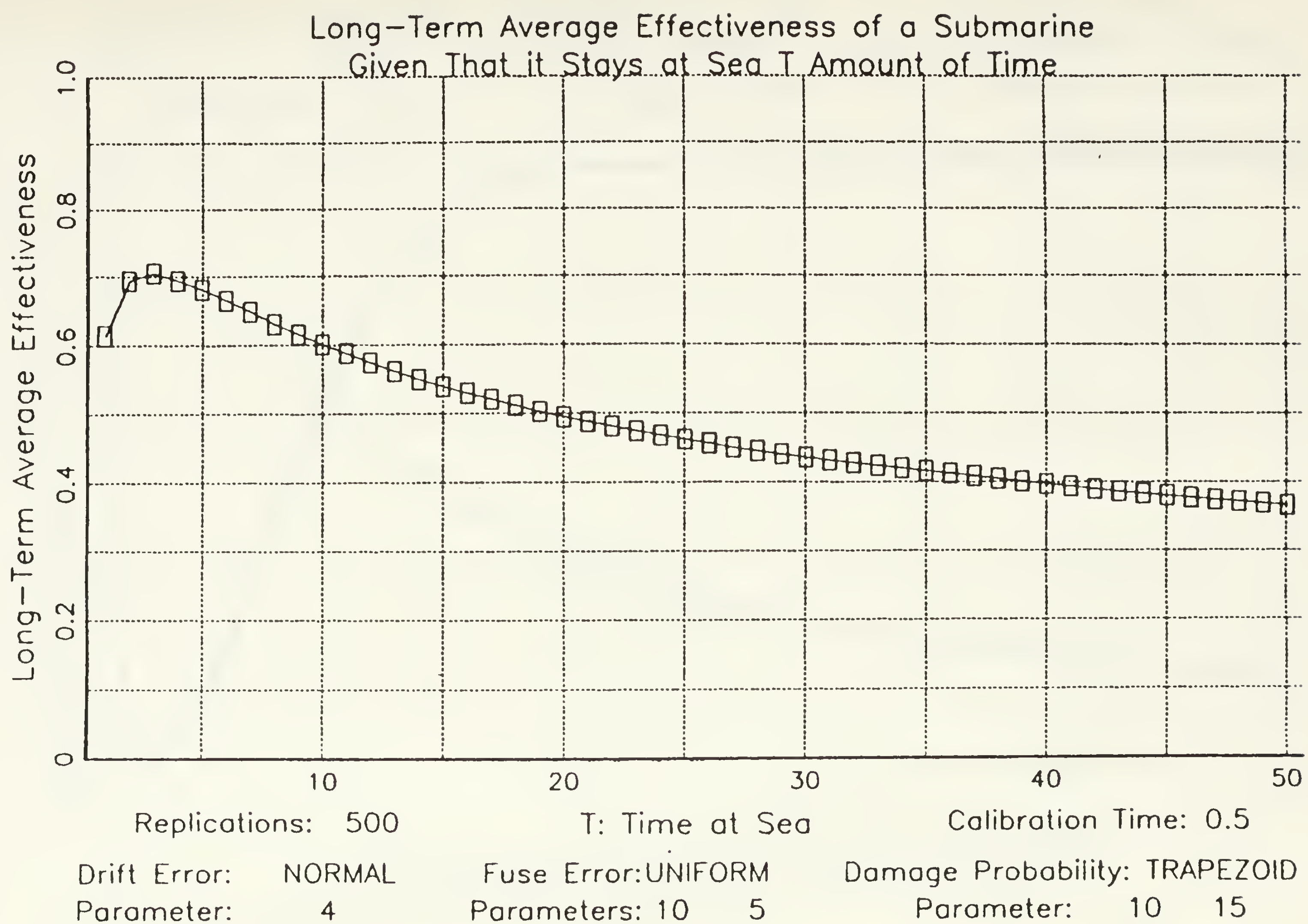
Replications: 500 T: Time at Sea Calibration Time: 0.5

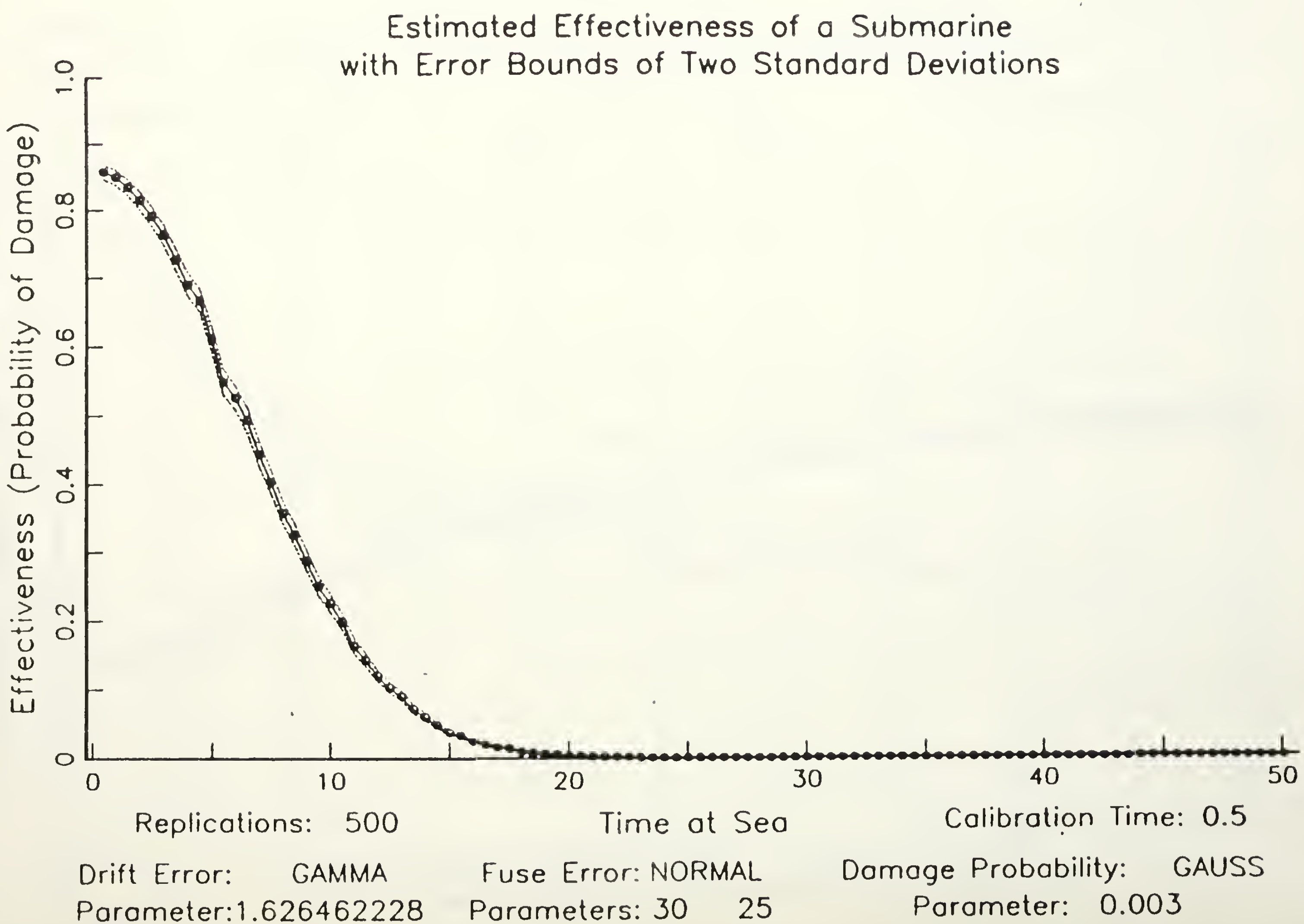
Drift Error: RANDOM Fuse Error: NORMAL Damage Probability: GAUSS
Parameter: 3 Parameters: 25 15 Parameter: 0.0015

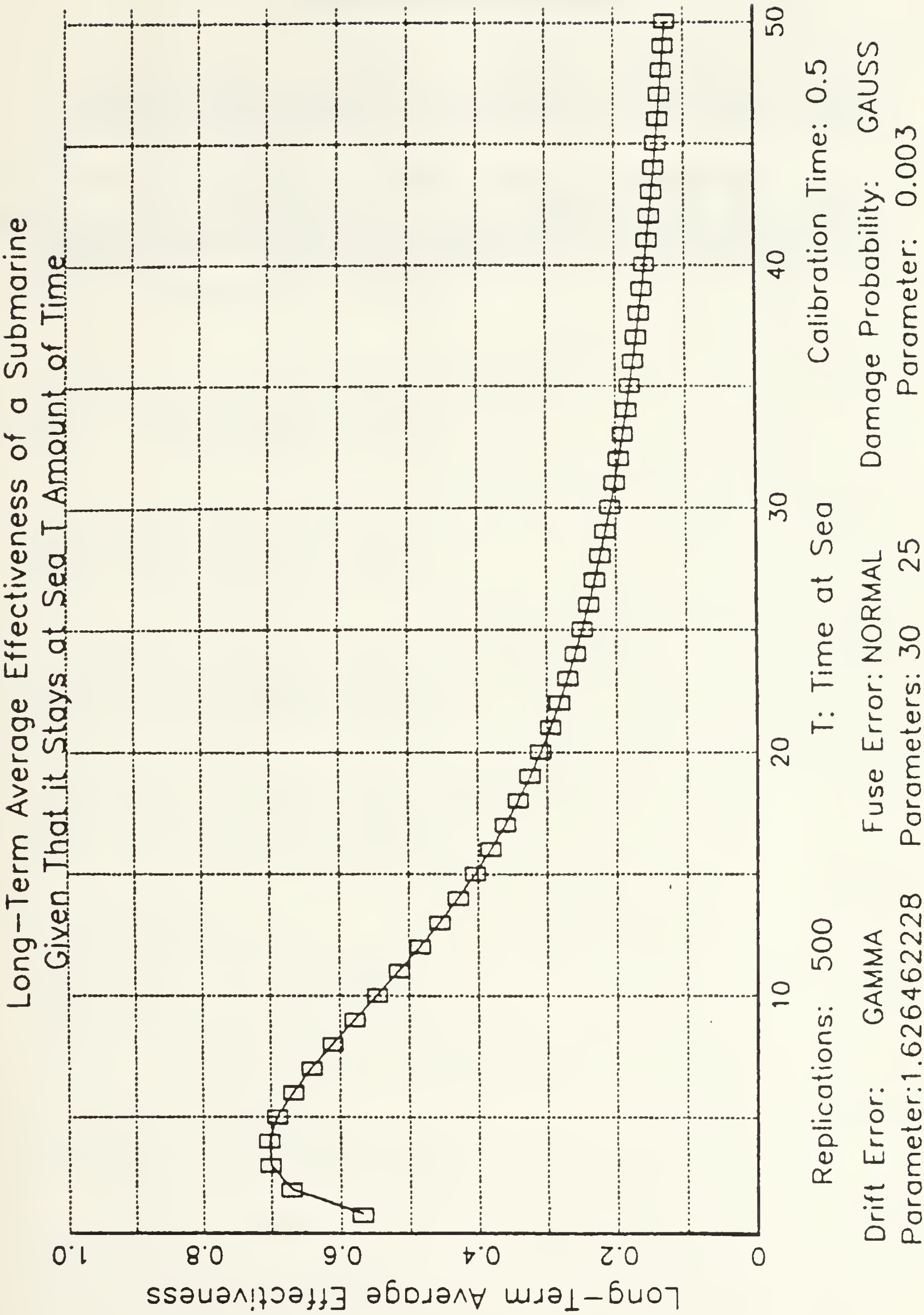


Replications: 500 Calibration Time: 0.5

Drift Error:	NORMAL	Fuse Error:	UNIFORM	Damage Probability:	TRAPEZOID
Parameter:	4	Parameters:	10 5	Parameter:	10 15


Long-Term Average Effectiveness of a Submarine
Given That it Stays at Sea T Amount of Time
Long-Term Average Effectiveness
0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
T: Time at Sea
Replications: 500
Calibration Time: 0.5
Drift Error: NORMAL
Parameter: 4
Fuse Error:UNIFORM
Parameters: 10 5
Damage Probability: TRAPEZOID
Parameter: 10 15


Estimated Effectiveness of a Submarine
with Error Bounds of Two Standard Deviations
Effectiveness (Probability of Damage)
0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
Replications: 500
Time at Sea
Calibration Time: 0.5
Drift Error: GAMMA
Parameter:1.626462228
Fuse Error: NORMAL
Parameters: 30 25
Damage Probability: GAUSS
Parameter: 0.003


Long-Term Average Effectiveness of a Submarine
Given That it Stays at Sea T Amount of Time
Long-Term Average Effectiveness
0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
T: Time at Sea
Replications: 500
Calibration Time: 0.5
Drift Error: GAMMA
Parameter:1.626462228
Fuse Error: NORMAL
Parameters: 30 25
Damage Probability: GAUSS
Parameter: 0.003

LIST OF REFERENCES

1. Eckler, A. Ross, and Burr, Stefan A., Mathematical Models of Target Coverage and Missile Allocation, MORS, 1972.

2. Naval Postgraduate School, Technical Report NPS-62-82-041 PR, Fleet Operational Readiness Strongly Influenced by the Calibration Condition of Surface Ship Weapon Systems, Stentz, D. A., May 1982.

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